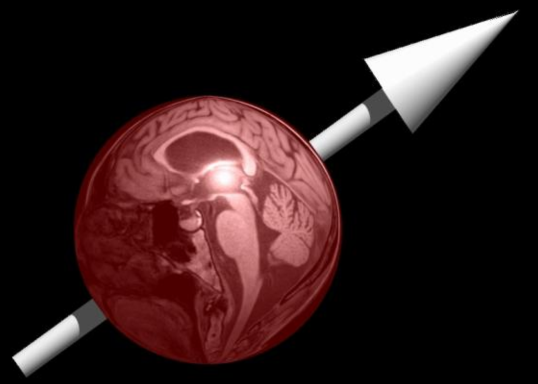
The Physics of Magnetic Resonance Imaging (MRI) Lecture 1



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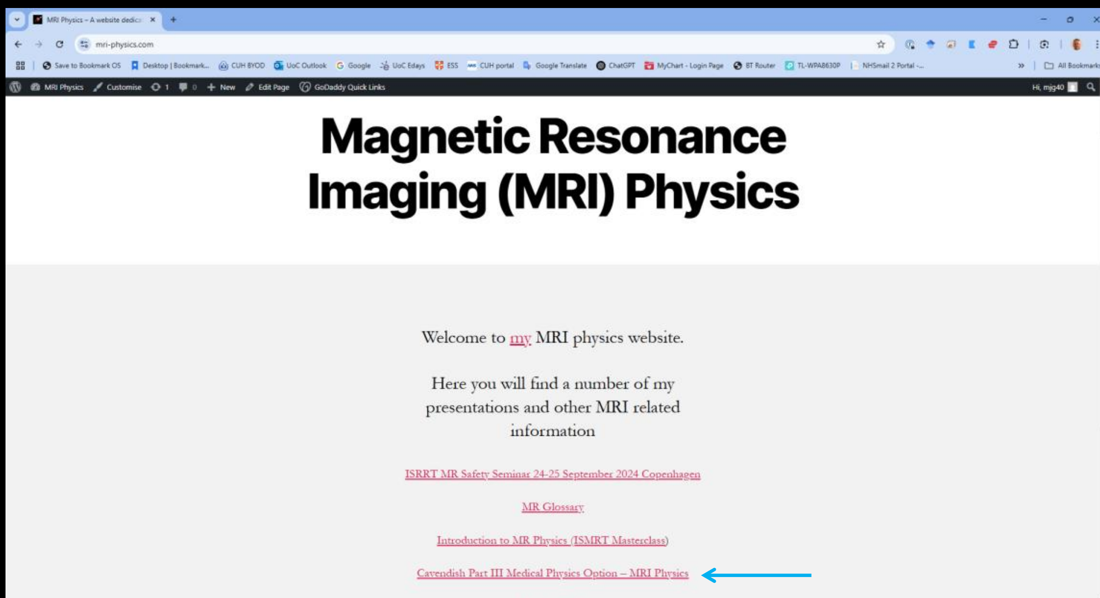
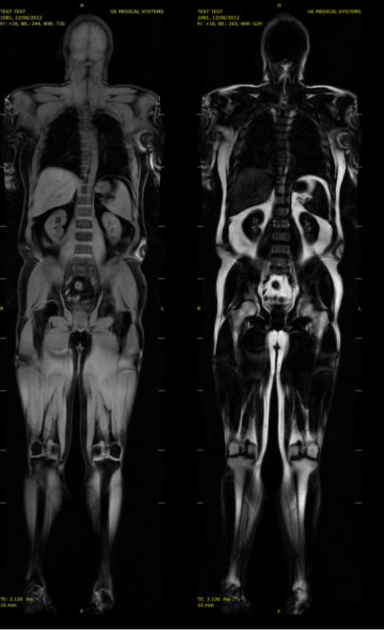


Learning Outcomes

- After these three lectures you should be able to:
  - Explain how nuclear spin gives rise to magnetic resonance
  - Understand the principles of T_1, T_2 and T_2^*relaxation
  - Explain the principles of MR image formation
  - Describe the spin echo and gradient echo pulse sequences
  - Outline the basic components of an MRI system
  - Understand the safety issues related to MRI

Magnetic Resonance Imaging

- Based upon nuclear magnetic resonance (NMR)
- MRI primarily looks at the nucleus of hydrogen
  - Human body 60% water and 16% fat
- MRI uses strong magnetic fields
  - Protons behave as microscopic bar magnets
- Protons resonate with external magnetic field
  - The net magnetisation can be manipulated



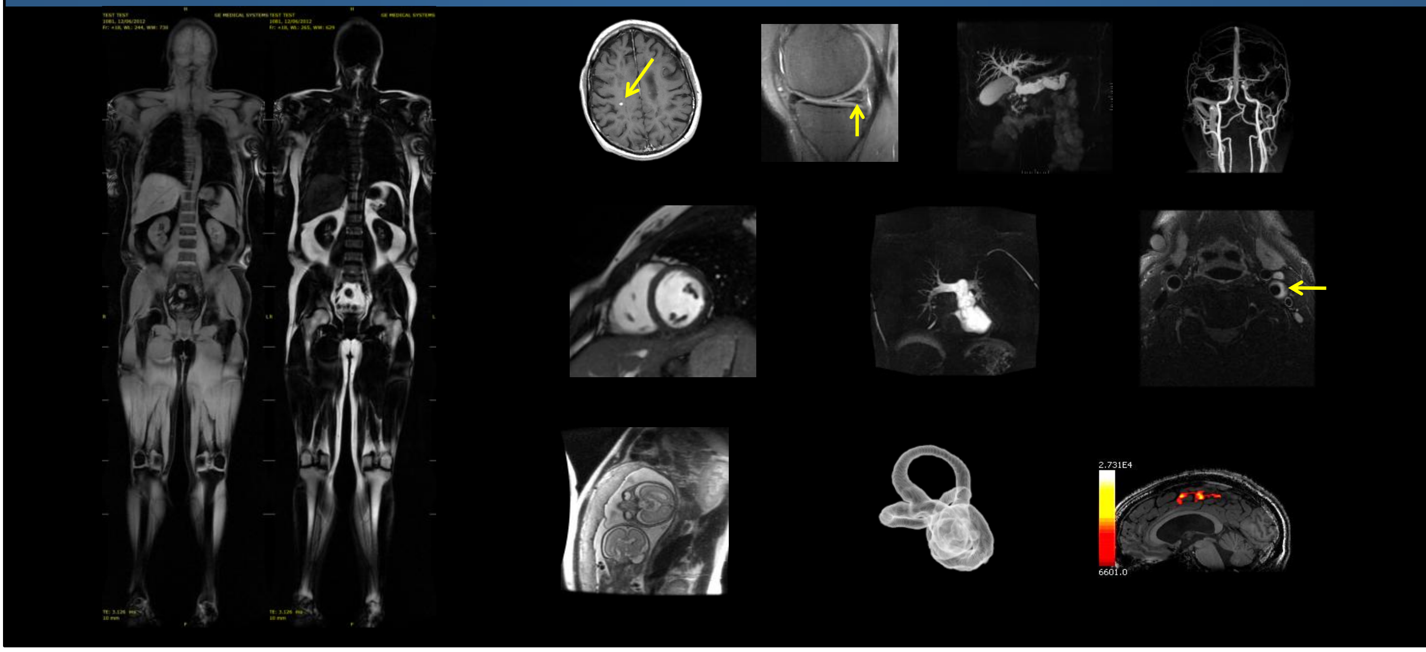
Briefly magnetic resonance imaging is based upon the phenomenon of nuclear magnetic resonance (NMR). If we look at the various terms that make up NMR. Firstly, MRI primarily looks at the nucleus of hydrogen. A typical adult is approximately 60% water and 16% fat so it is possible to create images which are sensitive to the protons within water and fat molecules although other NMR active nuclei can also be imaged in certain circumstances. MRI uses strong magnetic fields; therefore, the protons have a magnetic moment which can interact with the magnetic fields of the MRI system to create images. These hydrogen protons can be considered to precess at a specific frequency and therefore they can interact with a matching or a resonant external alternating magnetic field which can be used to manipulate the magnetization.

Clinical Applications

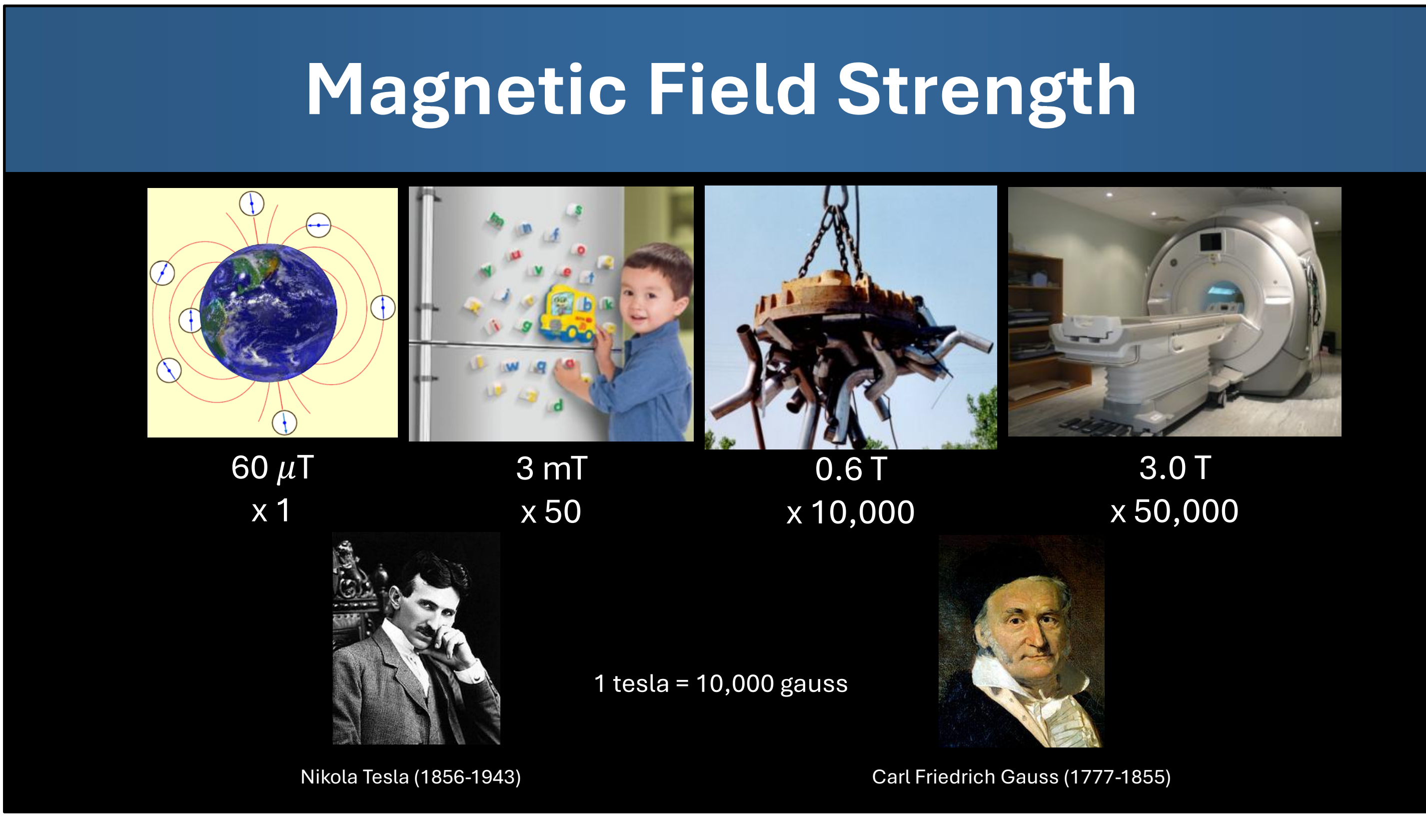
- Neurology – stroke, multiple sclerosis, tumours
- Oncology – diagnosis, staging, treatment response
- Musculoskeletal – trauma, degeneration
- Cardiovascular – heart and blood vessel morphology and function
- Infectious diseases and inflammation
- Congenital/anatomical defects

MRI has a wide range of clinical applications. In neurology it can be used to investigate stroke, multiple sclerosis, and brain tumours. In general oncology it is used for the diagnosis, staging, and looking at the response of cancers treatment. In the musculoskeletal system it can be used to investigate trauma and joint degeneration. In the cardiovascular system, i.e., the heart and blood vessels MRI can be used to look at both the morphology and the structure of the heart as well as the cardiac function. It is also widely used for infectious diseases and inflammation and looking at congenital or anatomical defects.

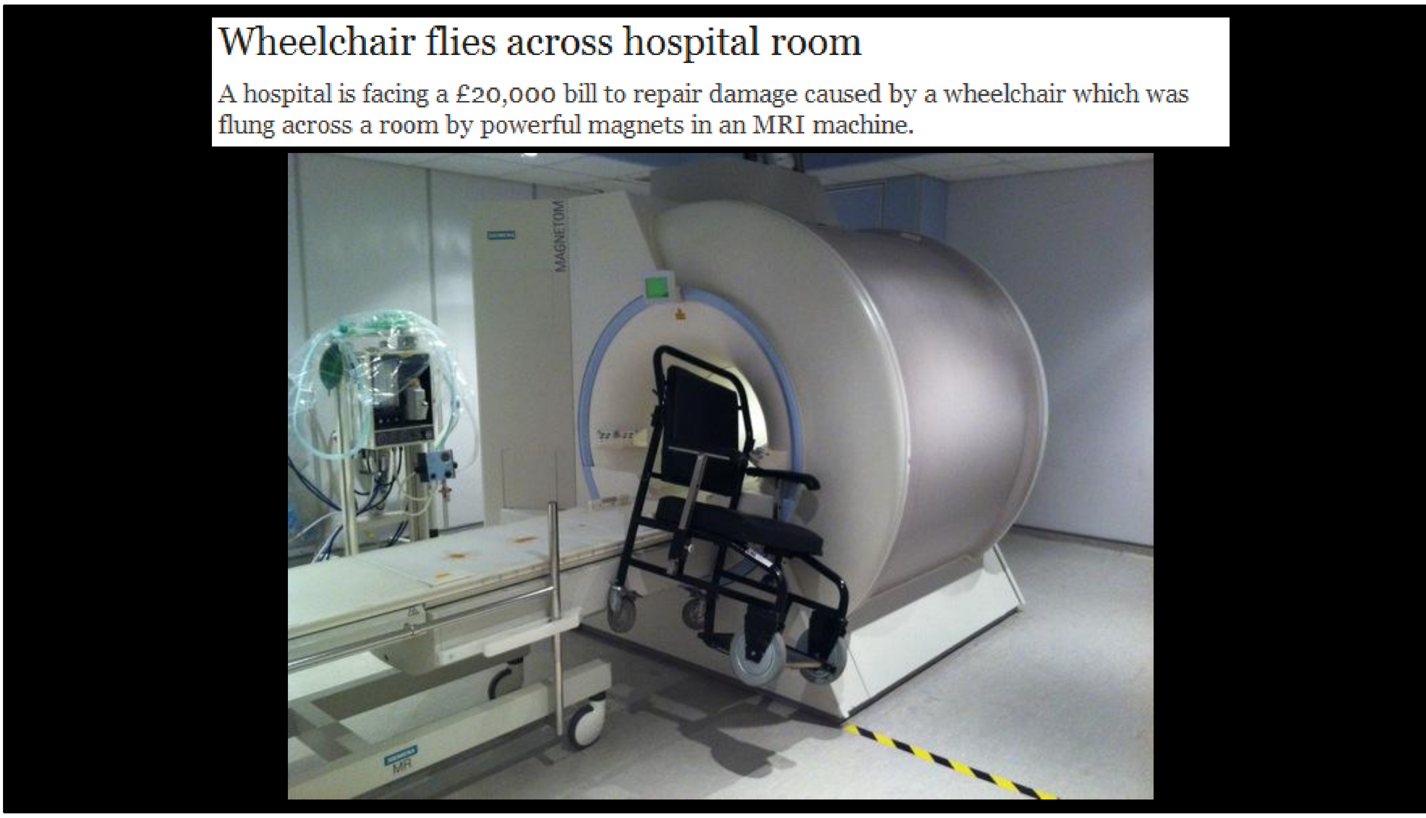
Example Clinical Applications



Here are some example MRI clinical applications. On the left we have a whole-body MRI scan representing the distribution of the protons in water and next to it an image showing the distribution of the protons in fat. The montage of images starting at the top left we have a section through the brain and the arrow is pointing towards an active multiple sclerosis plaque which appears very bright due to the administration of an MRI contrast agent. Next is a musculoskeletal example in the knee. The femur is at the top and the tibia at the bottom with the meniscus between them. The arrow is pointing to a torn meniscus. Next is an example of a 3D magnetic resonance cholangiopancreatography (MRCP) scan which looks at the bile ducts. Next is a 3D magnetic resonance angiogram that shows the blood vessels within the neck and the intra cranial circulation. The vessels are bright simply because the protons in blood happen to be moving. The middle row firstly shows an example of a cine cardiac MRI scan where you can see the heart contracting during the cardiac cycle. This acquisition takes place over several heartbeats. The imaging plane is in what's known as the short axis view of the heart where the bright signal is the blood and the darker signal around the blood is the left ventricular myocardium. The dark regions inside the left ventricular blood pool are the papillary muscles. To the left is the right ventricle. The acquisition is synchronised to the patient’s ECG so as the heart contracts you can clearly see the motion and thickening of the myocardium. Next is an example of a dynamic contrast enhanced 4D (three spatial + time) magnetic resonance angiogram this is where we inject a bolus of an MRI contrast agent, and you can see how it passes through the blood vessels in a dynamic fashion. First, we see the enhancement of the pulmonary circulation and then the arterial circulation followed by the venous return Next is an example from some of our research work where we've been looking at acquiring very high-resolution images of the carotid artery wall which is a potential area for the formation and potential rupture of atherosclerotic plaques that can give rise to thromboembolic strokes. The bright signal pointed to by the arrow represents a very thick fibrous cap around a lipid core which is part of the atherosclerotic plaque surrounding that vessel. The final row shows an example of fetal MRI, where we image the fetus in utero. This is pregnancy with twins, and we can see very nicely demonstrated the brains of the two babies. Next is another example of very high-resolution MRI imaging. Here we have segmented the images to show the semi-circular canals in their three planes and the cochlear. Finally, there is an example of functional magnetic resonance imaging where the colour overlay on the image represents the change in blood flow that is associated with the subject tapping their fingers and activating their motor cortex.



We are all familiar with the fact that the earth has a magnetic field, that is the reason why compass needles point almost North and we can measure magnetic field strength in units of either gauss (G), named after the child prodigy Carl Friedrich Gauss, or the SI unit being the tesla (T) named after Nikola Tesla the famous Serbian American inventor. The earth's magnetic field is approximately 60 mT. Fridge magnets probably have a field strength of around 3 mT. The magnets that you might see in a junkyard for moving heavy iron objects are around 0.6 T. To generate such a large magnetic field requires the use of the electromagnet where the magnetic field is created by an electric current flowing through a wire wrapped many times around an iron core. Finally, the magnets used in the NHS for MRI are typically either 1.5 T or 3 T. This is a picture of the 3 T MRI scanner that we have in the radiology department in Addenbrooke’s.



The very strong static magnetic field associated with clinical MRI systems pose a serious safety risk with any ferromagnetic object be it scissors or in this case a wheelchair turning into a projectile due to the attractive force of the magnet. Fortunately, nobody was injured in this case for this wheelchair would have been attracted to the magnet at around 30 mph.

A Brief History of NMR & MRI

- Based on the principle of Nuclear Magnetic Resonance discovered by Bloch and Purcell *simultaneously* in 1946 (awarded 1952 Nobel Prize in Physics)
- The initial concept for the medical application of NMR originated with Damadian in 1971
- NMR imaging of two tubes of water first demonstrated by Lauterbur in 1973
- Slice selection (and other things) invented by Mansfield in 1973
- First commercial system developed by EMI in 1975
- 2003 Nobel Prize for Physiology or Medicine award to Lauterbur and Mansfield for "their discoveries concerning magnetic resonance imaging"


Edward Purcell (1912-1997)


Felix Bloch (1901-1999)


Raymond Damadian (1936-)

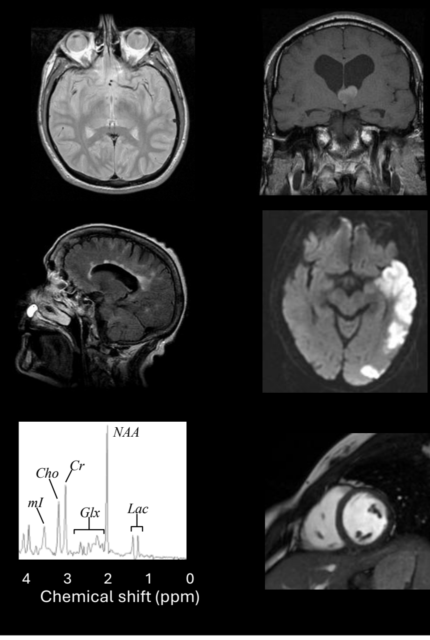

Peter Mansfield (1933-2017)


Paul Lauterbur (1929-2007)

MRI is based on the principle of nuclear magnetic resonance that was discovered by Felix Bloch working at Stanford and Edward Purcell working at Harvard almost simultaneously in 1946. They were subsequently awarded the 1952 Nobel Prize in physics for "the development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith". The Boston Herald reported that Purcell's discovery 'wouldn't revolutionize industry or help the housewife'. Bloch, a Swiss-born Jew, and friend of quantum physicist Werner Heisenberg, quit his post in Leipzig in 1933 in disgust at the Nazi's expulsion of German Jews (as a Swiss citizen, Bloch himself was exempt). Bloch's subsequent career at Stanford was crammed with major contributions to physics and he has been called the father of solid-state physics. The initial concept for the medical application of MRI originated with Raymond Damadian in 1971. He and his colleagues at the State University of New York, who were starved of mainstream research funding, even went so far as to design, and build their own superconducting magnets operating in their Brooklyn laboratories. The first human NMR image is attributed to them. NMR imaging of two tubes of water was first demonstrated by Paul Lauterbur in 1973. He is said to have been inspired to use field gradients to produce an image whilst eating a hamburger! His seminal paper "Image formation by induced local interactions, examples employing nuclear magnetic resonance" (Nature 242, March 16, 1973) was originally rejected. However, 30 years later Nature placed this work in a book of the 21 most influential scientific papers of the 20th century. In the UK, a lot of work in MRI was performed primarily at the universities of Aberdeen and Nottingham. Slice selection, amongst other things, being invented by Professor Sir Peter Mansfield at the University of Nottingham in 1973. The first commercial MRI system was developed by EMI in 1975 in the UK, primarily based on money that they had earned from The Beatles albums. In 2003 the Nobel Prize for Physiology or Medicine was jointly awarded to Lauterbur and Mansfield "for their discoveries concerning magnetic resonance imaging ". This was much to the chagrin of Raymond Damadian who also felt that he should have been included. There will always be some dispute about who is the inventor of modern MRI, but one thing is certain, Damadian coined the first MR acronym, namely FONAR (Field fOcused Nuclear mAgnetic Resonance) for his original painfully slow method of creating MRI images, voxel-by-voxel.

MRI Advantages

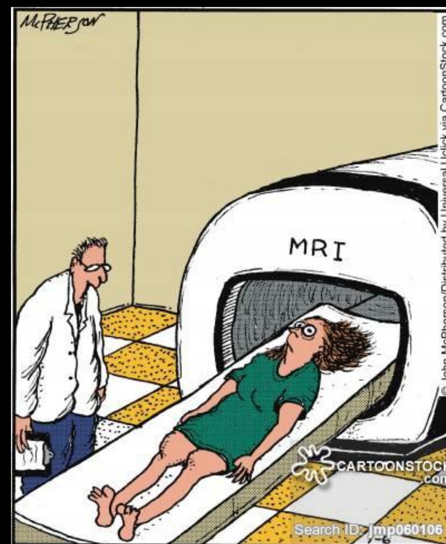
- No ionising radiation
  - Magnetic fields only
- Multi-planar
  - Axial, coronal, sagittal, single/double oblique
- Multiple contrast mechanisms
  - Proton density, T_1, T_2, flow, diffusion, chemical exchange, spectroscopy …
- Good spatial resolution
  - Variable field-of-view
- Reasonable temporal resolution
  - Dynamic/functional imaging



Unlike X-ray or nuclear medicine-based imaging methods MRI does not utilise ionizing radiation. Instead, it relies on magnetic fields for image formation. MRI is a tomographic imaging technique that acquires data in multiple thin slices or sections. These sections can be acquired in any orientation, including oblique planes. MR images can reflect several different contrast mechanisms including proton density, T_1 and T_2 relaxation times and number of other microscopic and macroscopic phenomena such as diffusion. MRI is particularly noted for its excellent soft-tissue contrast. Spectroscopy can be used to investigate different tissue metabolites since the protons within these molecules exhibit small but detectable chemical shifts

MRI Limitations

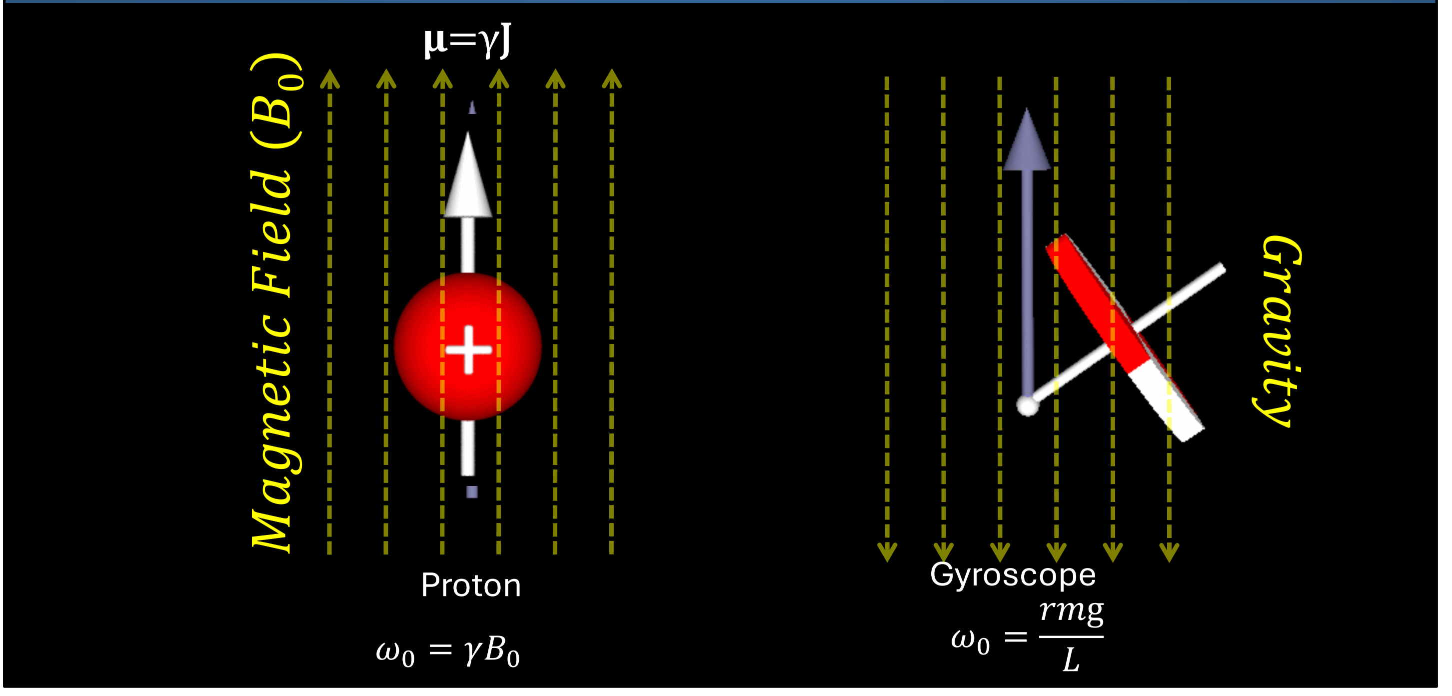
- Strong magnetic field
  - Safety, acoustic noise
- Relatively slow
  - Compared to X-rays, ultrasound, CT
- Cannot usually directly visualise calcium
  - Compared to X-rays and CT
- Complex technology
  - Expensive, requires skilled operators (Radiographers)
- Subjects can feel claustrophobic (bore diameter 60/70cm)



"OK, Mrs. Dunn. We'll slide you in there, scan your brain, and see if we can find out why you've been having these spells of claustrophobia."

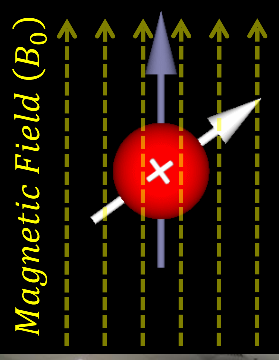
MRI has several limitations compared to other medical imaging techniques. Since it utilises very strong static magnetic fields, we have already seen the projectile risk from ferromagnetic objects coming near to the magnet. The gradients that are used to spatially localise the MRI signal also physically flex like a loudspeaker creating acoustic noise. MRI is also relatively slow compared to x-rays, ultrasound, or CT examinations. A typical MRI examination will comprise several imaging series each producing a different contrast and often in different imaging planes. Whilst we can acquire some MRI images in under a second most clinical imaging series take several minutes to acquire the data. A complete MRI examination can take anything from around ten minutes to an hour depending upon the complexity of the investigation. Standard MRI sequences also cannot visualise cortical bone, hence x-ray CT is still the method of choice for trauma imaging due to its speed and its ability to visualise bone. The wide range of imaging parameters makes MRI a complex and expensive technology that requires skilled operators or radiographers to perform the examinations. In addition, most MRI systems are based on superconducting magnet technology which has typically a 60 or 70 cm diameter patient aperture, which can make some patients feel claustrophobic

Spin

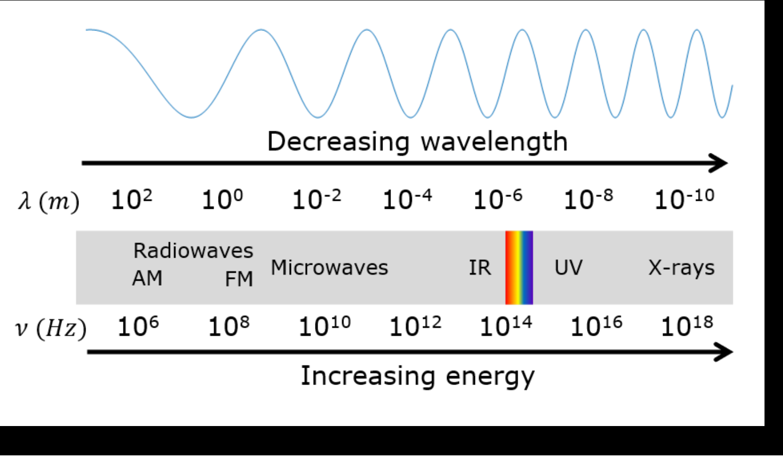


For the purposes of this discussion, we will take a fanciful and relativistically inadequate representation of the proton as a spinning ball of charge. Due to the combination of charge and spin we can consider that the proton behaves as a tiny bar magnet and has a nuclear magnetic moment **m**. The ratio of the magnetic moment **m** to the angular momentum **J** is called the gyromagnetic ratio g. When placed in a static magnetic field B_0 we can consider that mu makes an angle with respect to the direction of B_0. It therefore experiences a torque and precesses about B_0 at a characteristic frequency $\omega_0 = -\gamma B_0$. The minus sign, which will get quietly dropped in future, just means that defines a clockwise rotation about B_0. The proton procession is analogous to the procession of a gyroscope in the earths gravitational field. Where r is the distance from the pivot m is the mass, g is acceleration due to gravity and L is the angular momentum.

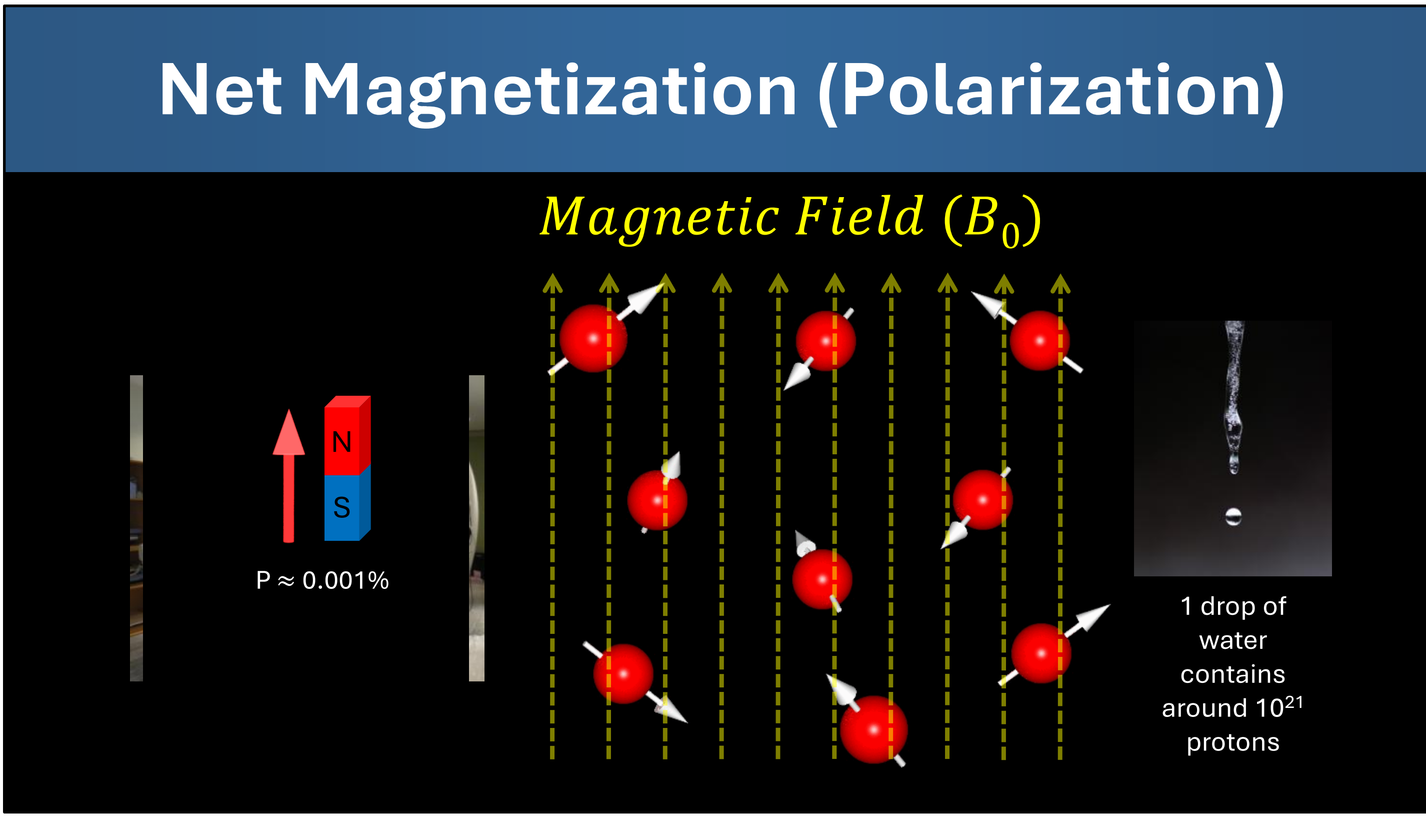
Precessional Frequency



For [1]H $\gamma = 42.57$MHz/T
i.e., at 3.0T the precessional frequency of [1]H is 128 MHz (RF)

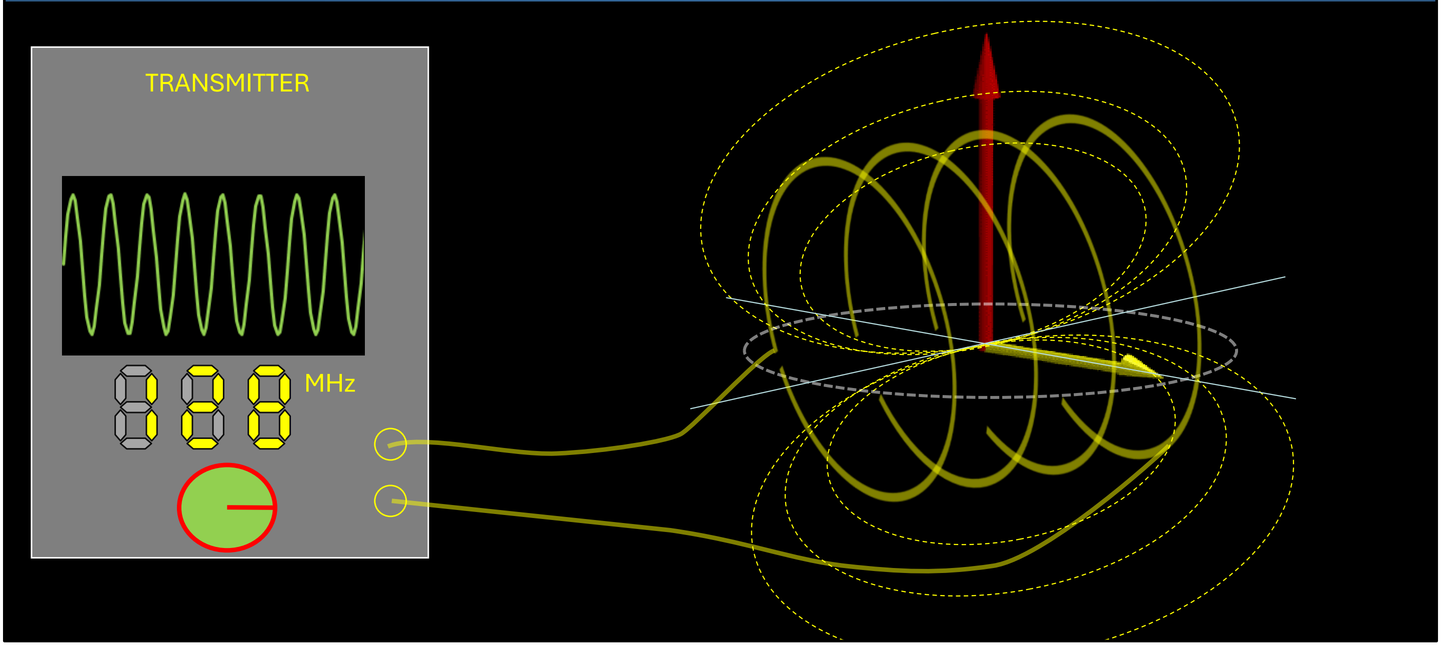


As we have seen the precessional frequency is dependent upon the strength of B_0 and in our 3 T magnet the hydrogen nucleus, which has a gyromagnetic ratio of 42.57 MHz/T will precess at a frequency of 128 MHz, which is in the radio frequency part of the electromagnetic spectrum and is therefore a much lower frequency than those associated with X-rays. Hence why MRI is considered as a non-ionizing imaging modality.



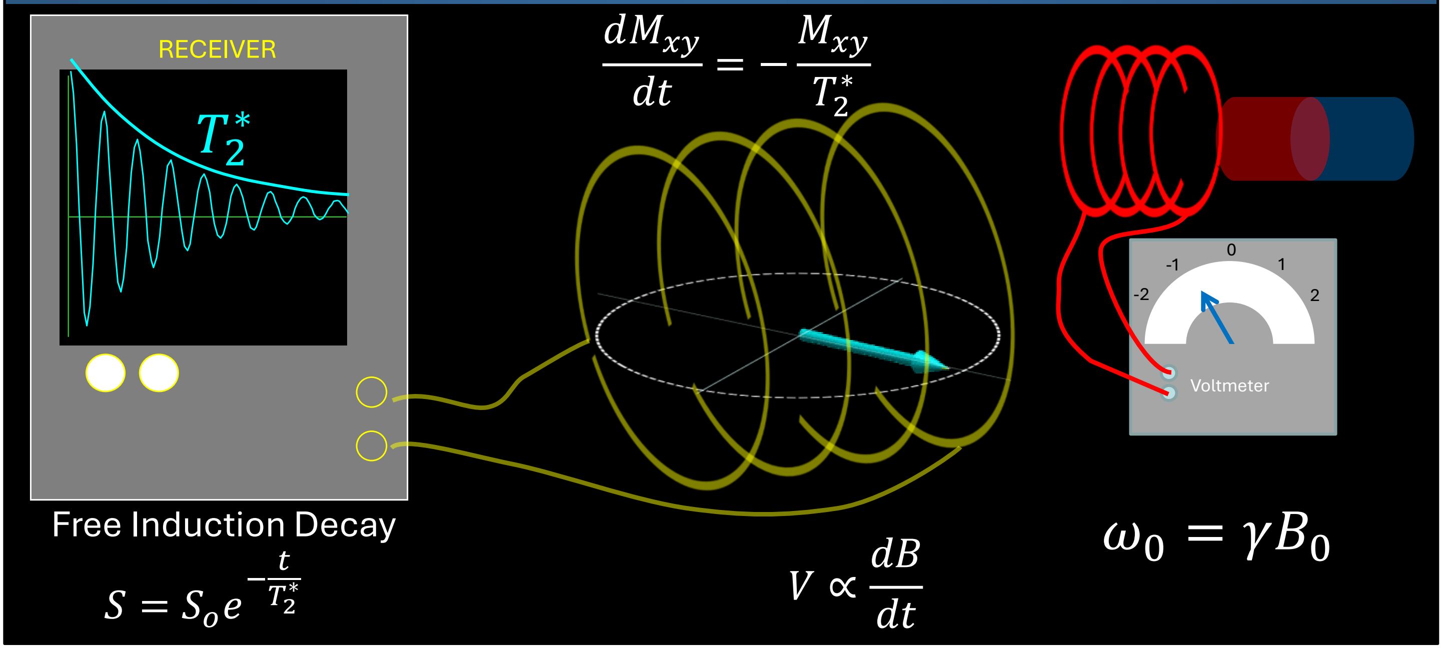
In addition to the individual protons precessing around the direction of B_0 the protons also become distributed into one of two states: either parallel to B_0 or antiparallel. The probability distribution means that just over half of the protons end up pointing in the same direction as B_0 and just under half end up pointing in the opposite direction. This is known as the nuclear polarisation (P). We can consider the ensemble of those pointing up as a large bar magnet pointing in the same direction as B_0 whilst the ones pointing down appear as a slightly smaller bar magnet pointing in the opposite direction. Since they are pointing in opposite directions, they subtract from each other leaving a small net magnetization pointing in the same direction as B_0. Even though this polarisation is a very small percentage, approximately 0.001% at 3 T and half that value at 1.5 T there are a very large number of protons in the human body, giving rise to a small but detectable net magnetisation aligned with B_0.

RF Excitation & Precession



Since the net magnetization **M** (red arrow) is pointing in the same direction as B_0 it is effectively undetectable and therefore needs to be aligned orthogonal to B_0. We will denote the z-direction as the direction of the net magnetization at equilibrium. We can perturb **M** away from z by applying a second weaker magnetic field called B_1 (yellow arrow) at right angles to B_0. If the B_1 field is alternating at the same frequency as the natural precessional frequency of the proton, i.e., it is resonant with ω_0 then in the laboratory frame of reference the net magnetization will spiral down for as long as B_1 is applied. The simplest arrangement is to apply an alternating current to a solenoid thereby creating an alternating magnetic field inside the solenoid at right angles to B_0. In this example B_0 is 3 T, so the alternating field is at 128 MHz

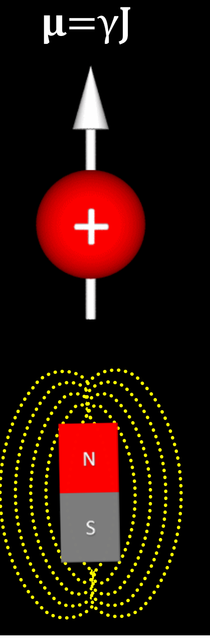
Signal Reception



Once **M** has been tipped through 90° and we turn off B_1 then **M** will continue to precess in the transverse plane and, according to Faraday's Law of induction, this precession will induce an emf in the same solenoid that was originally used to create the B_1 field. Due to small variations in the uniformity of the B_0 magnetic field the net transverse magnetisation (cyan arrow) will decay in amplitude whilst precessing, and the induced emf can be seen on the oscilloscope as an exponentially decaying sinusoid. This is known as a free induction decay and is the most basic NMR signal. The time constant of the decay is characterised by a value known as T_2^*. The small positive and negative variations in B_0 means that some nuclei precess slightly faster (yellow), and some precess slightly slower (white) than the nominal ω_0 so the net transverse magnetization decreases in amplitude as the nuclei dephase with respect to each other in the transverse plane

Principles of NMR

- Protons possess an intrinsic angular momentum **J** and are positively charged
- A moving charge generates a magnetic field
- Hence, we can consider a proton to behave like a bar magnets, i.e., it has a magnetic moment **μ**
- The magnetic moment **μ** is given by $\boldsymbol{\mu}=\gamma\mathbf{J}$, where γ is the gyromagnetic ratio



The physical basis of nuclear magnetic resonance NMR centres around the concept of a nuclear spin, its associated angular momentum, and its magnetic moment. Spin is a purely quantum mechanical quantity with no easy classical analogy despite the simplified representation used here. For a hydrogen nucleus, i.e., a single proton the spin angular momentum **J** and the magnetic moment **m** are related through the proportionality constant γ known as the gyromagnetic ratio which is a fundamental constant of a nucleus.

Nuclear Spin

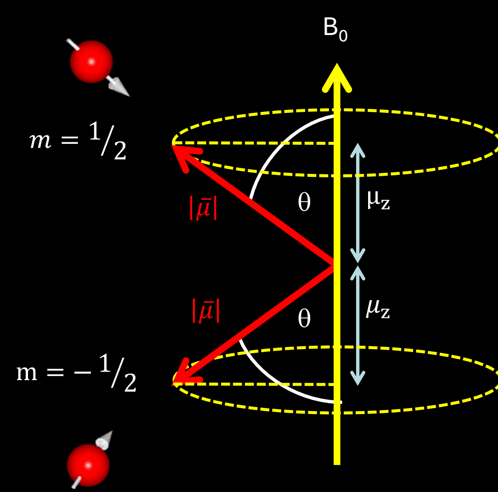
- The magnitude and direction of **J** is characterized by the nuclear spin quantum number I
- Nuclei with an equal number of protons and neutrons have zero spin
  - $I = 0$, e.g., ^{12}C, ^{16}O
- If the number of neutrons **plus** the number of protons is odd, then the nucleus has half-integer spin
  - $I = {}^{1}/_{2}$, e.g., ^{1}H, ^{19}F, ^{13}C, ^{31}P, ^{15}N
  - $I = {}^{3}/_{2}$, e.g., ^{23}Na, ^{37}Cl
- If the number of neutrons **and** the number of protons are both odd, then the nucleus has integer spin
  - $I = 1$, e.g., ^{2}H, ^{14}N

The magnitude and direction of the spin angular momentum J is characterised by the nuclear spin quantum number I. Nuclei with an equal number of protons and neutrons have zero spin. If the number of neutrons plus the number of protons is odd, then the nucleus has half integer spin and if the number of neutrons and the number of protons are both odd then the nucleus has integer spin. The vast majority of clinical MRI involves the use of hydrogen nuclei in water or fat. Since the nucleus of hydrogen is a single proton, it has a spin quantum number of one half.

Nuclear Splitting in an External Magnetic Field (B_0)

- $\boldsymbol{\mu} = \gamma \mathbf{J} = \hbar\gamma\sqrt{I(I+1)}$
- $\mu_z = \gamma J_z = \hbar\gamma m_I$, where magnetic moment quantum number $m_I \in \{-I, \ldots, I-1, I\}$ for a total of $2I+1$ values
- For nuclei with $I = {}^1/_2$, e.g., ^1H there are two orientations with $m_I = {}^1/_2$ and $-{}^1/_2$
- *Zeeman observed spectral line splitting in an external magnetic field in 1896*

Pieter Zeeman
1865-1943



In 1896 Pieter Zeeman observed the splitting of optical spectral lines in an external magnetic field. Since then, the splitting of energy levels proportional to an external magnetic field has been called the Zeeman effect, with the effect in the nucleus known as the Nuclear Zeeman effect. For protons with a spin quantum number of ½ there are two orientations with the magnetic moment quantum numbers of ± ½. Note that $\hbar$ is the reduced Planck's constant equal to ($\hbar = \frac{h}{2\pi}$).

The vectors z projection is given by $\hbar m_I$ (where h = 6.63x10^{-34} J s) and where m_I is the magnetic moment quantum number and has values of $-I \leq m_I \leq I$

For nuclei with $I = \frac{1}{2}$

The angle of **J** is given by

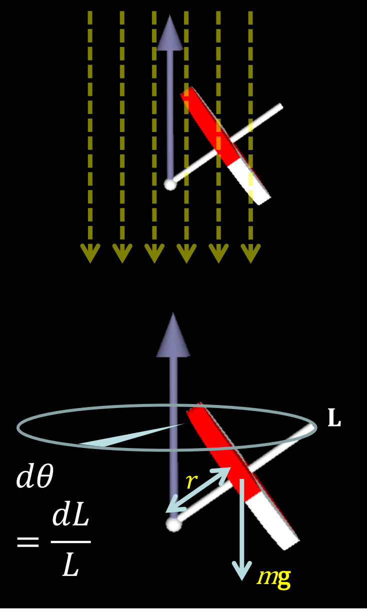
$$|\bar{\mu}| = \hbar\gamma[I(I+1)]^{1/2}$$

$$\mu_z = \pm\frac{1}{2}\hbar\gamma$$

The angle is therefore $\cos^{-1}\left(\frac{\mu_z}{|\bar{\mu}|}\right) = \frac{1/2}{\sqrt{\left(\frac{1}{2}\left(\frac{1}{2}+1\right)\right)}} = \frac{1/2}{\sqrt{3/4}} = 0.544 = 54.7°$

Gyroscope Precession

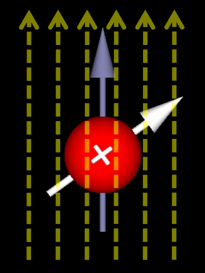
- Torque $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} = \mathbf{r} \times \mathbf{F}$
- $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} = \mathbf{r} \times m\mathbf{g}$, since $\mathbf{F} = m\mathbf{g}$
- $d\theta = \frac{dL}{L} = \frac{rmg}{L} dt$
- $\omega_0 = \frac{d\theta}{dt} = \frac{rmg}{L}$



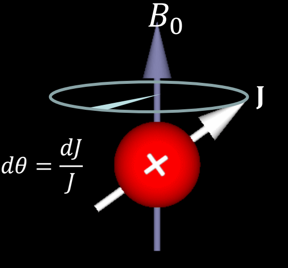
We have already introduced the analogy of nuclear spin to that of a gyroscope, where gravity will cause the gyroscope to experience a torque and therefore precess about the vertical. Here I simply derive the precessional frequency for a gyroscope of mass M distance r, angular momentum L and g is the standard acceleration due to gravity.

Classical Description of Nuclear Precession

- Torque $\boldsymbol{\tau} = \frac{d\mathbf{J}}{dt} = \boldsymbol{\mu} \times \mathbf{B}$
- $\boldsymbol{\tau} = \frac{d\mathbf{J}}{dt} = \gamma\mathbf{J} \times \mathbf{B}$, since $\boldsymbol{\mu} = \gamma\mathbf{J}$
- $d\theta = \frac{dJ}{J} = \gamma B_0 dt$
- $\omega_0 = \frac{d\theta}{dt} = -\gamma B_0$
- $f_0 = \frac{\gamma}{2\pi} B_0 = \bar{\gamma} B_0$
- *In 1897 Larmor proposed that charged particles should precess about a magnetic field and the frequency of precession should be directly proportional to the field strength*



Joseph Larmor
1857-1942



Similarly, to the gyroscope the static magnetic field will cause our spinning nuclei to experience a torque and hence process around B_0. This precessional frequency is given by the Larmor equation $\omega_0 = \gamma B_0$. Larmor worked in an era when the basic structure of the atom was still being discovered and quantum mechanics had yet to be developed. His famous equation of 1897 was unrelated to NMR as this phenomenon was still several decades away from being discovered. Instead, it arose from attempt to explain the Zeeman splitting observed one year previously. Larmor's theory was that Zeeman's spectral lines were produced by charged particles moving in elliptical orbits. Larmor demonstrated mathematically that these particles should precess around the direction of the applied magnetic field. He further calculated that the frequency of precession was directly proportional to the strength of the applied field times a constant. Larmor's constant was directly related to the particles charge to mass ratio. In later years, as more became known about atomic structure, Larmor's equation was found to apply to any particle with spin or angular momentum taking the form we recognise today. The Larmor frequency is a signed quantity and is negative for nuclei with a positive gyromagnetic ratio. This means that for such spins the precession frequency is negative as shown in the animation. Note that conventionally the Larmor equation is written $\omega_0 = \gamma B_0$, where ω_0 is the angular frequency of the protons precession which is 2.67x10^8 rad/s/T, however this number is rather unmemorable and as angular frequencies are not as intuitively understandable as regular scalar frequencies, I prefer to refer to γ as 42.57 MHz/T, that is $\frac{\gamma}{2\pi}$, sometimes referred to as $\bar{\gamma}$ (gamma bar).

Some NMR Active Nuclei

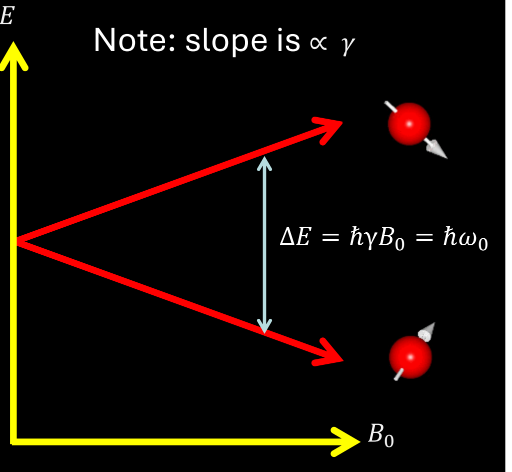
Nucleus	Unpaired Protons	Unpaired Neutrons	Net Spin	γ (MHz/T)	Natural Abundance (%)
^{1}H	1	0	1/2	42.58	99.98
^{13}C	0	1	1/2	10.71	1.1
^{14}N	1	1	1	3.08	99.6
^{19}F	1	0	1/2	40.08	100
^{23}Na	1	2	3/2	11.27	100
^{31}P	1	0	1/2	17.25	100

Here is a table of some common NMR active nuclei, together with their gyromagnetic ratios. Note that the neutron has spin ½ but no net charge. This can be reconciled with the classical model if the neutron is considered as a proton orbited by a negatively charged pion. The neutron is composed of three quarks and the magnetic moments of these elementary particles combine to give the neutron its magnetic moment.

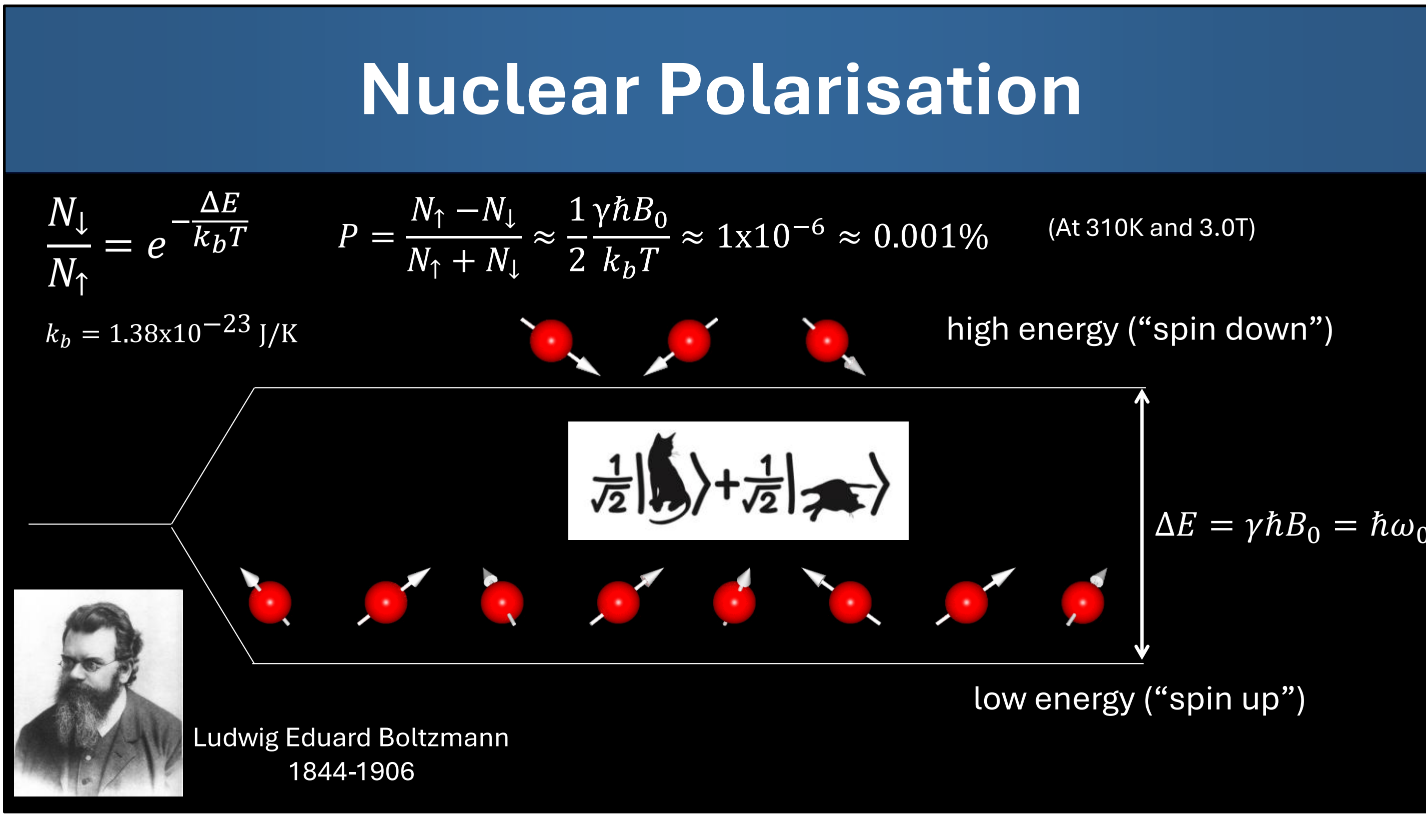
Energy Level Splitting

- The potential energy E of a magnetic moment $(\boldsymbol{\mu})$ in a magnetic field $\mathbf{B}$ is given by
  - $E = -\boldsymbol{\mu} \cdot \mathbf{B} = \hbar\gamma m_I \mathbf{B}$
- Selection rules only allow for transitions between $-I \leq m_I \leq I$ so for $I = {}^1/_2$
  - $\Delta E = ({}^1/_2 - -{}^1/_2)\gamma\hbar B_0 = \hbar\gamma B_0$
- From De Broglie's wave equation
  - $\Delta E = \hbar\omega$
- $\hbar\gamma B_0 = \hbar\omega_0$
- $\gamma B_0 = \omega_0$

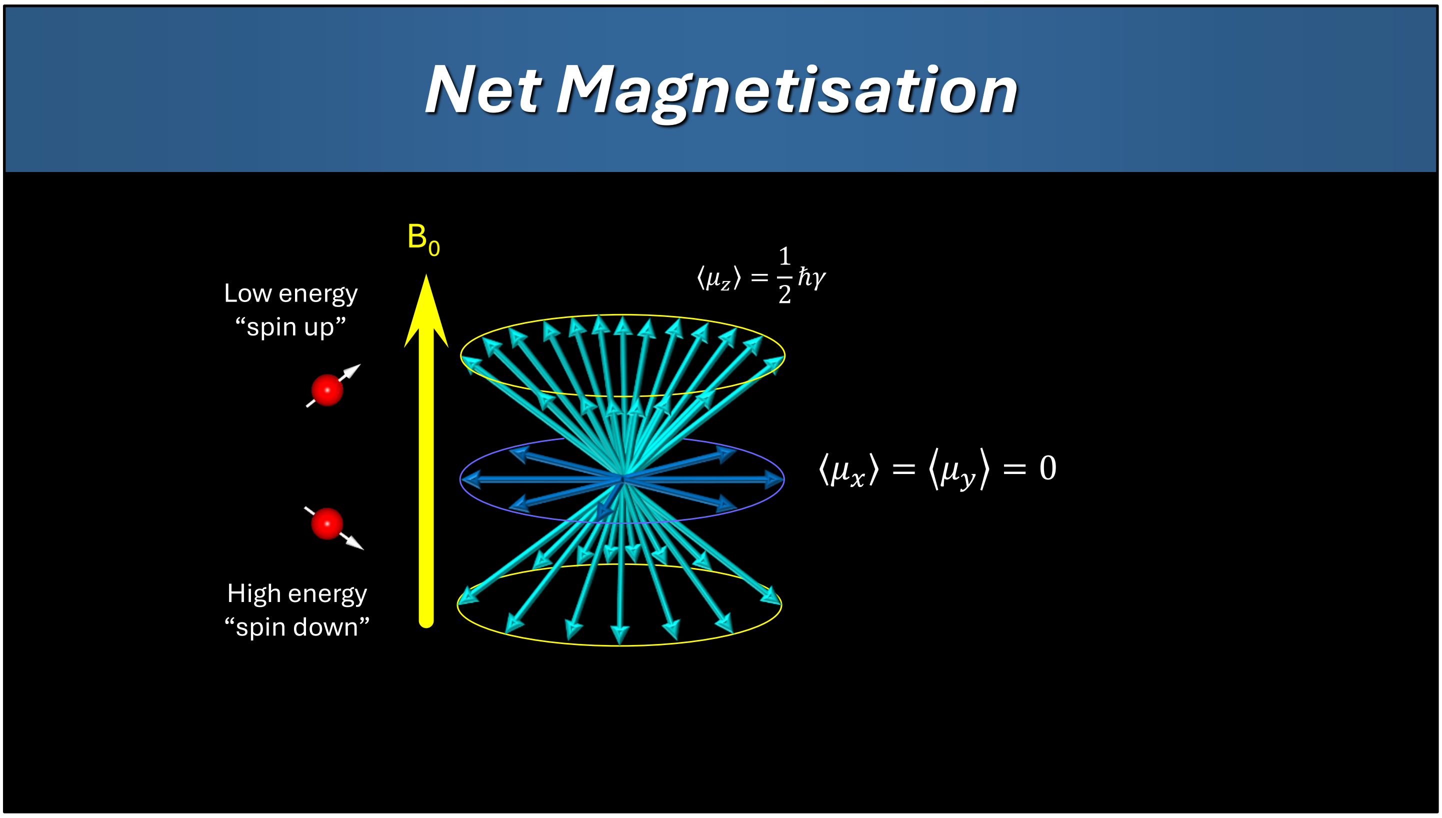
Louis de Broglie
1892-1987



From a quantum mechanical perspective, the energy difference between the two orientations in a static magnetic field of B_0 is given by $\Delta E = \gamma\hbar B_0$ and is exactly equivalent to the classical Larmor equation $\omega_0 = \gamma B_0$.

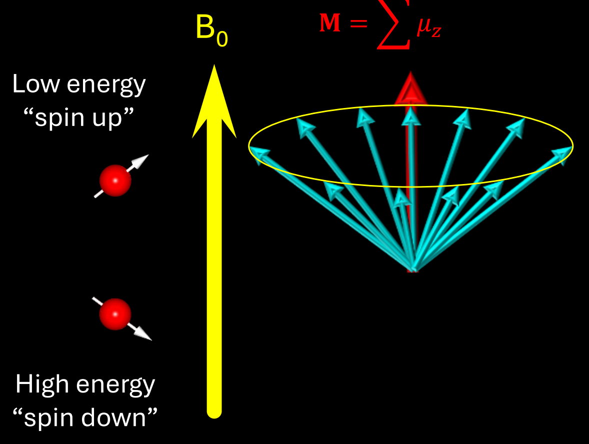


The population distribution known as the polarisation (P) of the nuclei between the two states is characterised by the Boltzmann distribution which describes the distribution of energy among classical or distinguishable particles. Where $k_b$is Boltzmann’s constant (1.38x10^{-23} J/K) and T is the absolute temperature (K) (human body temperature is normally about 310K). If we inject energy into this system at an angular frequency of ω_0, we can induce transitions between the two energy states. A spin-up nucleus can absorb energy and transition to a spin down state and a spin-down nucleus can give up energy and transition to spin-up. The excitation must be at this specific frequency to “resonate” with the nuclei as described previously. The cartoon of Schrodinger’s cat is to remind us that the spin and magnetic moment exist in all directions simultaneously, but their average behaviour is non-zero in only one of the directions. Therefore, these diagrams are an attempt to provide some additional insights into how NMR works but their limitations should be appreciated.



The spin (and associated magnetic moment and angular momentum) is probabilistic in nature (much in the same way that electrons surrounding the nucleus travel in probabilistic shells). Thus, each spin does not really align with B_0, but rather exists in a probabilistic cone and spin-up and spin-down implies that the cone faces up or down. The spin and magnetic moment exist in all directions simultaneously, but the average behaviour is non-zero in only one of the directions. Each spin contributes $\frac{1}{2}\hbar\gamma$ to the overall z-magnetisation of the sample. We cannot observe individual spins, only the ensemble average **M**, that we refer to as the net magnetisation

Net Magnetisation



$$M_0 \approx \frac{\rho_0 \gamma^2 \hbar^2 B_0}{4 k_b T}$$

k_b - Boltzmann's constant
$\hbar$ - reduced Planck constant
T - temperature in K

Water contains 6.7×10^{22} protons ml^{-1}. At normal body temperature at 3.0 T we get $M_0 \approx 0.002\ \mu T ml^{-1}$. Assuming the human head has volume of 1500 ml and is approximately 80% water we get $M_0 \approx 2\ \mu T$

Molar mass H_2O = 18 g/mol

1 g H_2O = 1 g / 18 g/mol = 0.0556 mol H_2O

1 mol H_2O = $6.022*10^{23}$ molecules

0.0556 mol H_2O = 0.0556 mol * $6.022*10^{23}$ molecules / mol = $3.35*10^{22}$ molecules

1 g H_2O contains $3.35*10^{22}$ molecules

Each H_2O molecule contains 2 protons (from ^2H) and 8 protons (from O)
Therefore 1 g H_2O contains : $3.35*10^{22}$ molecules * 2 protons / molecule = $6.7*10^{22}$ protons.

The individual transverse components cancel leaving only the z-components. The equation gives us an approximate idea of the size of the equilibrium magnetization. The human head will therefore create a net magnetisation of around 2 μT at 3 T.

MR Signal

- To create an MR signal we need to excite the spins out of equilibrium, i.e., we must tip the net magnetization away from the B_0 direction
- A transmit coil will create an orthogonal magnetic field (B_1), alternating at the Larmor frequency that will rotate the net magnetization into the transverse (x-y) plane (RF pulse)
- The precessing transverse magnetisation will induce a voltage in a receiver coil
- After the pulse, the magnetization will return to thermal equilibrium by processes known as relaxation

This slide summarises the process required to create the NMR signal. We firstly need to excite the spins by tipping the net magnetization from along the z direction into the transverse plane. This is done by applying a short duration pulse of a voltage alternating at the Larmor frequency to a transmit coil that creates a uniform magnetic field B_1, orthogonal to B_0, inside the coil. The B_1 field rotates the net magnetization into the transverse plane. The processing and decaying transverse magnetization will then induce a voltage into the receiver coil. After the B_1 pulse, the magnetization will return to thermal equilibrium by processes known as relaxation.

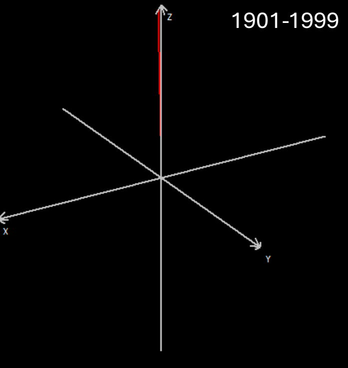
The Bloch Equation

- The dynamics of nuclear magnetization are described phenomenologically by the Bloch equation

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma\mathbf{B} - \frac{M_x i + M_y j}{T_2} - \frac{(M_z - M_0)k}{T_1} + \mathrm{D}\nabla^2 M$$

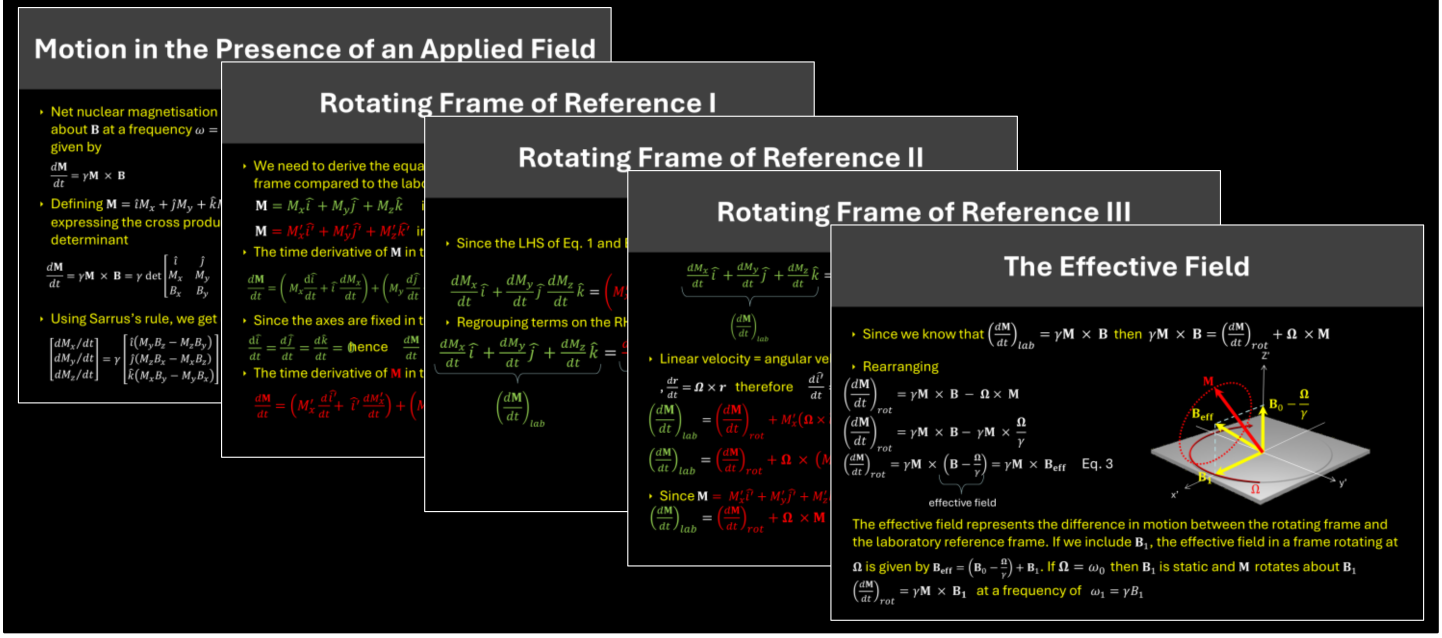
- The last term accounts for the transfer of magnetisation by diffusion, with D being the molecular self-diffusion coefficient
- The transverse component will dephase in the x-y plane with a time constant T_2. This is known as "spin-spin", or "transverse" or "T_2" relaxation. This process only involves energy exchange
- The longitudinal component will regrow along the z-direction with a time constant T_1. This is known as "spin-lattice", or "longitudinal" or "T_1" relaxation. This process involve energy loss as heat to the surrounding macromolecules aka "lattice"

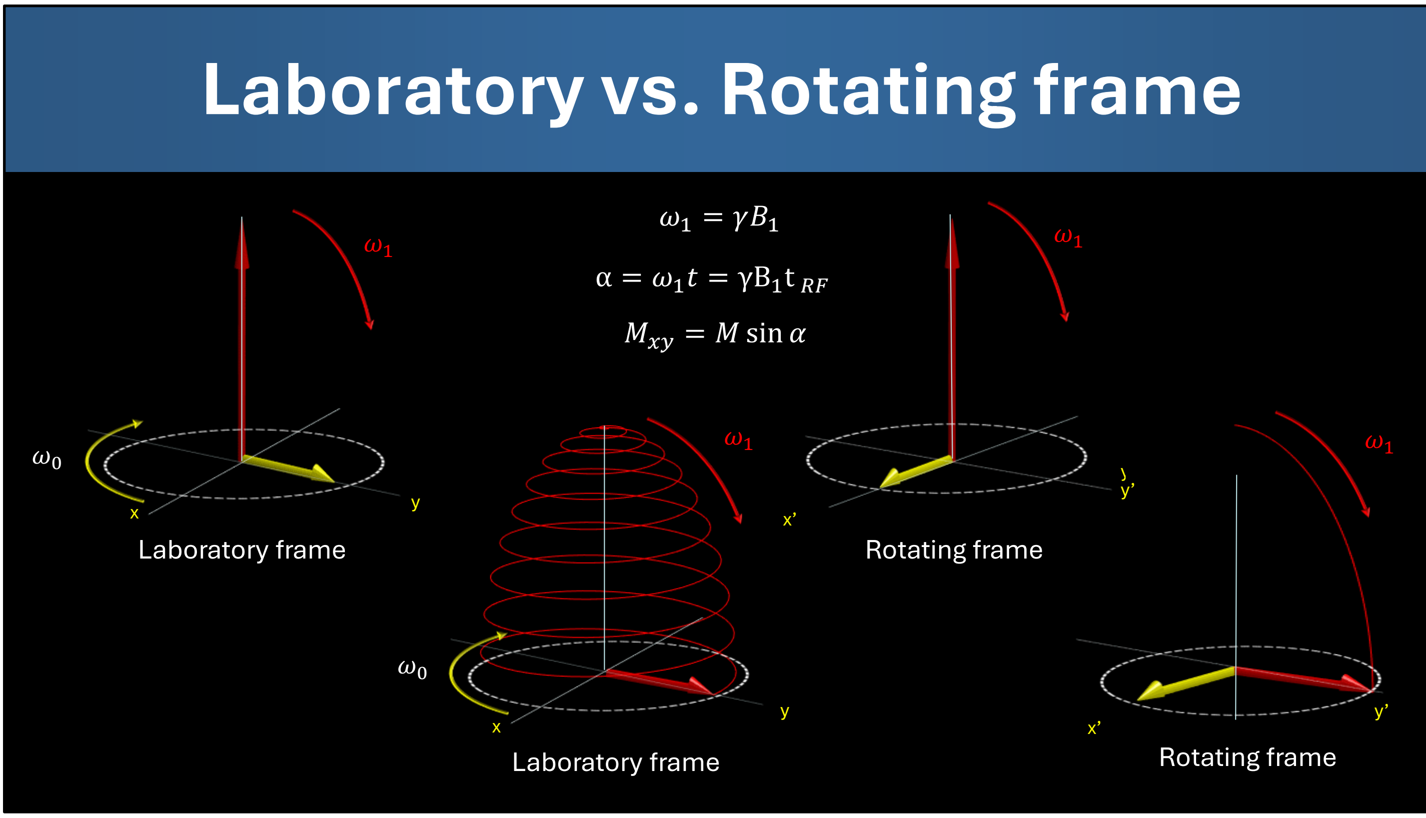
Felix Bloch
1901-1999



The Bloch equation describes the evolution of magnetization in a large static magnetic field considering spin relaxation. Bloch used the fact that an external field produces a torque or twisting force on the net magnetisation resulting in its precession at the Larmor frequency. He also reasoned that the decay of the NMR signal was due to the individual spins that comprised the net magnetization interacting with each other and their environment, a process Bloch termed relaxation. Bloch introduced two relaxation times, one to account for the recovery of the longitudinal magnetisation back to equilibrium (M_0), known as T_1, and one to account for decay of the transverse magnetization known as T_2. He also assumed that the decay of the transverse magnetisation and recovery of the longitudinal magnetisation was exponential. T_1 and T_2 were defined phenomenologically and not from fundamental principles although Bloch did correctly conclude that T_1 must result from thermal motion and T_2 from inter/intra -nuclear actions. Note in contrast to longitudinal decay, transverse decay conserves energy. The animation shows the net magnetization being tipped into the transverse plane followed by dephasing of the transverse magnetisation and recovery of the longitudinal magnetization. Note that the longitudinal recovery is longer than the transverse decay.

Additional (non-examinable info)





This slide shows the comparison of the motion in the laboratory frame and a frame rotating at the Larmor frequency. If the B_1 field shown as the yellow arrow is also rotating at the Larmor frequency, then it appears stationary in the rotating frame and the net magnetisation the red arrow simply rotates about the axis of the applied B_1 field (in this case the x-axis) at an angular frequency of γB_1. The time duration that the pulse is applied for will dictate the flip angle. In this case the pulse is applied for a sufficient time to tip the net magnetisation through 90° and hence such a pulse of RF is referred to as a 90° pulse. We will see the use of other RF flip angles later.

Bloch Equation Solution

- The Bloch equations with relaxation in the laboratory frame

$$\frac{dM_x}{dt} = \gamma(\mathbf{M} \times \mathbf{B})_z - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \gamma(\mathbf{M} \times \mathbf{B})_y - \frac{M_y}{T_2}$$

$$\frac{dM_z(t)}{dt} = \frac{(M_z - M_0)}{T_1} + \gamma(\mathbf{M} \times \mathbf{B})_z$$

- The solution for the case where $B = B_o$

$$M_x(t) = [M_x(0)\cos\omega_0 t + M_y(0)\sin\omega_0 t]e^{-t/T_2}$$

$$M_y(t) = [M_x(0)\cos\omega_0 t - M_y(0)\sin\omega_0 t]e^{-t/T_2}$$

$$M_z(t) = M_z(0)e^{-t/T_1} + M_0(1 - e^{-t/T_1})$$

- The equilibrium solutions are

$$M_x(\infty) = M_y(\infty) = 0 \qquad M_z(\infty) = M_0$$

- In the rotating frame of reference where $\omega = \omega_0$, and expressing the magnetisation in complex form $(M_{xy} = M_x + iM_y)$ the equations simplify

$$\frac{dM'_{xy}(t)}{dt} = -\frac{M'_{xy}}{T_2}$$

$$\frac{dM_z(t)}{dt} = -\frac{(M_z - M_0)}{T_1}$$

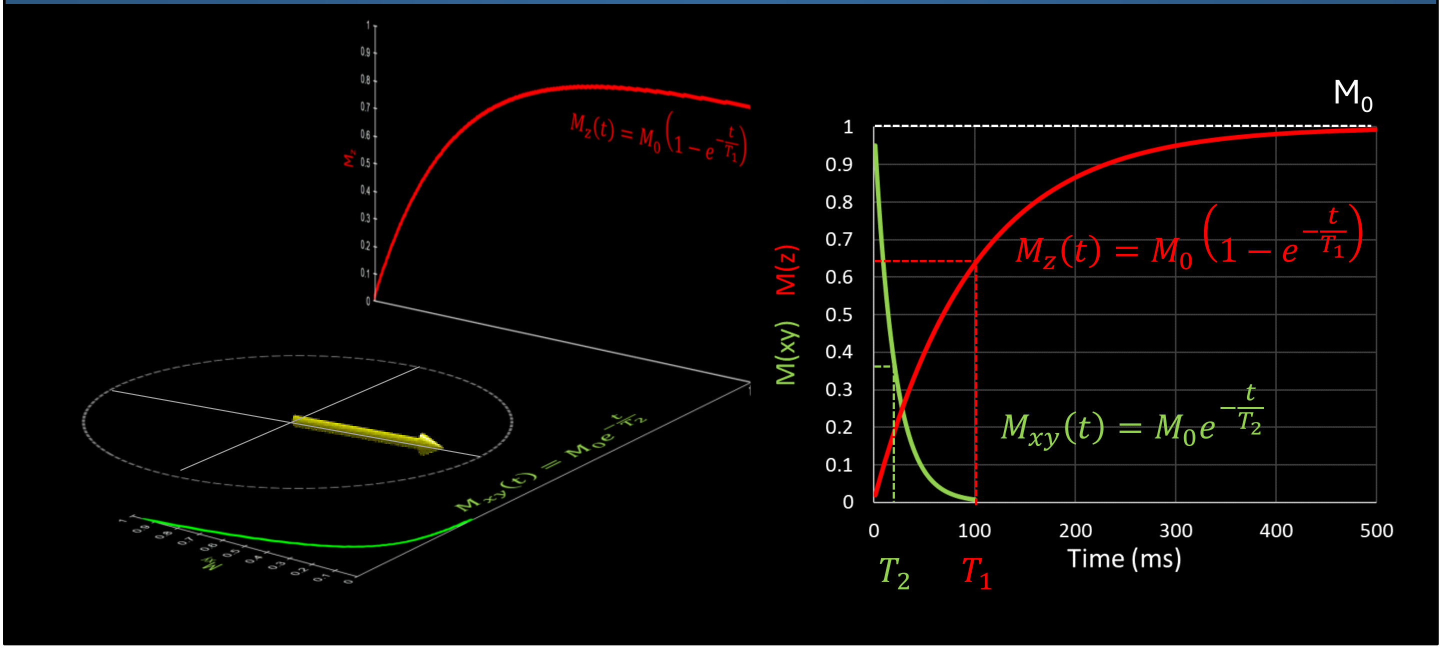
- These equations have the following solutions

$$M'_{xy}(t) = M_0 e^{-\frac{t}{T_2}}$$

$$M_z(t) = M_0\left(1 - e^{-\frac{t}{T_1}}\right)$$

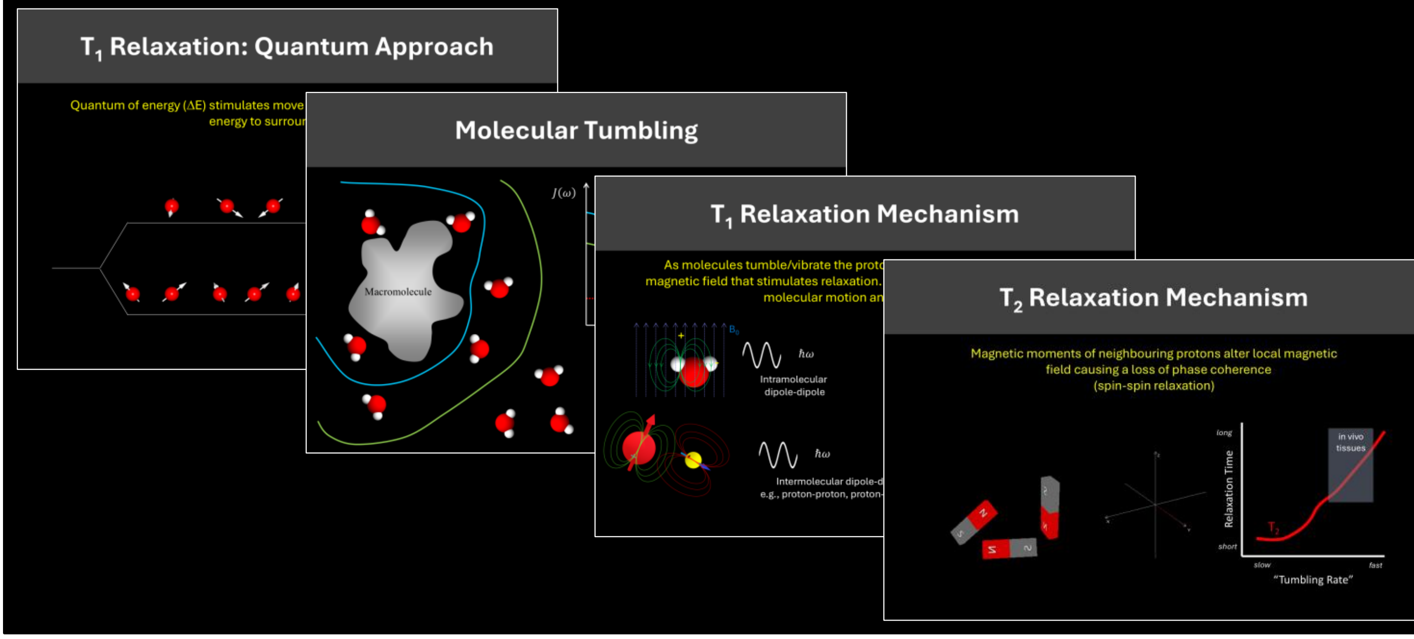
The solutions to the Bloch equations, with relaxation, in the laboratory and rotating frame are given here.

Transverse and Longitudinal Relaxation



Relaxation in the laboratory frame (yellow arrow) shows the magnetisation spiralling back to equilibrium due to T_2 and T_1 relaxation. In the rotating frame of reference, the transverse component (green arrow) decays due to T_2 relaxation and the longitudinal component (red arrow) recovers due to T_1. In biological tissue T_2 is approximately an order of magnitude shorter than T_1, i.e., for brain white matter the T_1 is around 780 ms and T2 is around 90 ms. Remember that M_0 is proportional to the proton density ρ.

Additional (non-examinable info)



RF Excitation

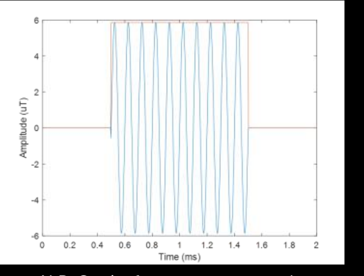
- An alternating magnetic field of amplitude $\mathbf{B_1}$ (μT) applied for time t_{RF} at the Larmor frequency ω_0 will tip $\mathbf{M}$ by $\alpha = \omega_1 t = \gamma B_1 t_{RF}$

E.g., a rectangular (hard) RF pulse of 1ms duration with a B_1 of 5.87μT will flip the magnetization by 90°. Note that B_1 is considerably smaller than B_0

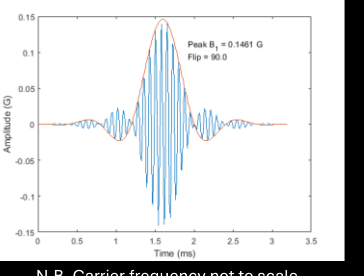
- RF pulses are the only way to rotate magnetization. Different flip angles are used for different applications
- In practice RF pulses will be shaped so more generally

$$\alpha = \gamma \int_0^{t_{RF}} B_1 dt$$

- Note that MRI uses alternating magnetic fields in the RF part of the EM spectrum. Note that it does not use radiowaves!



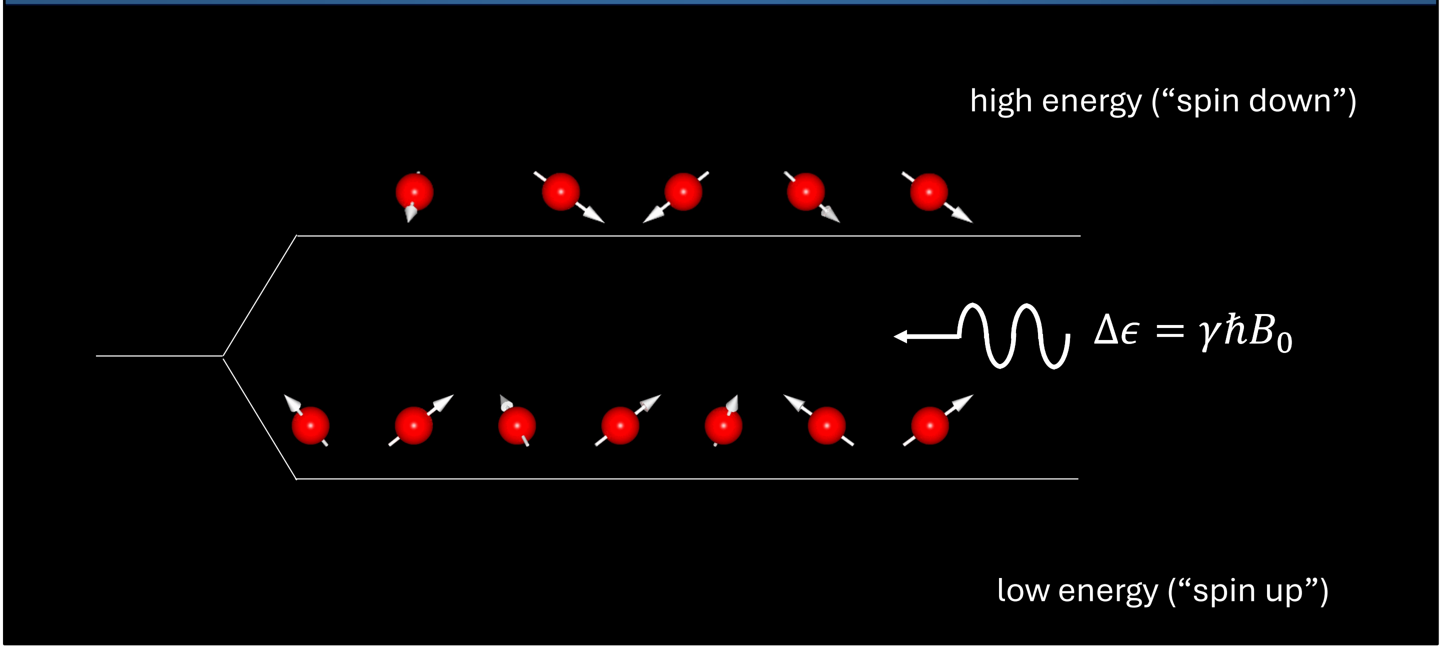
N.B. Carrier frequency not to scale



N.B. Carrier frequency not to scale

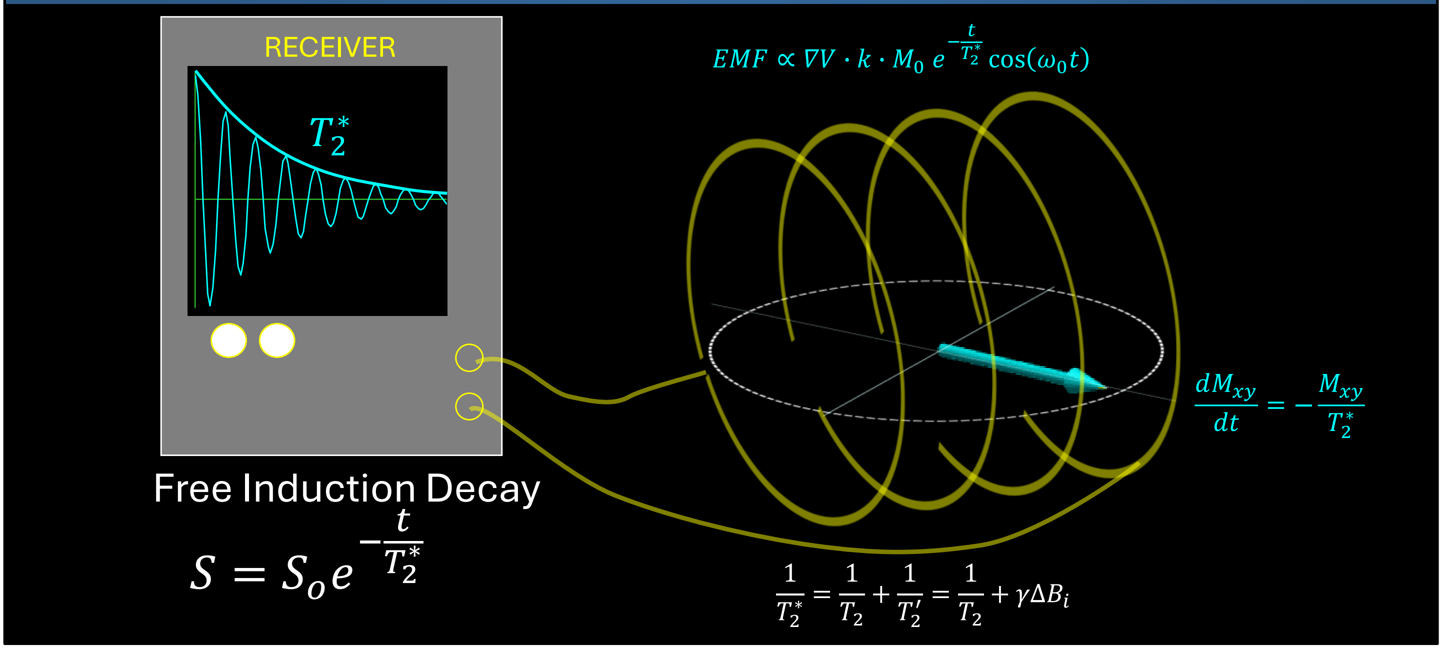
An RF pulses is alternating magnetic field (B_1) with an amplitude in terms of micro-tesla or gauss. The pulse is applied for a time t_{RF} at the Larmor frequency. A rectangular, or hard, RF pulse of duration 1 ms with an amplitude of 5.87 μT will tip **M** by 90°. Note that the amplitude of B_1 is considerably smaller than the static magnetic fields. RF pulses are the only way to rotate magnetisation and different flip angles get used for different applications. In practise RF pulses will be shaped, or soft, and the flip angle is going to be equal to the time integral of the B_1 envelope. A soft RF pulse is suitable for exciting a thin slice of spins. It is important to remember that MRI uses the alternating magnetic fields in the RF part of the electromagnetic spectrum - it does not use radio waves as is sometimes mentioned in the popular literature.

Excitation: Quantum Approach



Proceeding cautiously with our QM approach. When the population of protons is irradiated by an RF field, protons can flip between energy levels. Spin-up protons can absorb energy to jump into the spin-down position, while those in the spin-down state are stimulated into giving up an equal amount of energy to drop into the spin-up state, and there is an equal probability of each transition. Since in equilibrium there are more spin-up protons than spin-down, the net effect will be absorption of energy from the RF wave, causing the "temperature" of the spin system to rise. The protons temperature is considered separately from the temperature of the surrounding tissues, known as the lattice, which will eventually come into equilibrium with the spins. We will come back to this idea when we consider spin-lattice relaxation. Taking the simple idea of population difference and absorption of RF, the maximum absorption will be when all the spin-down protons have flipped into the spin-up position and vice versa. This is known as population inversion, and can be easily considered as a 180° pulse which flips the magnetization from z to -z. The definition of a 90° pulse can then be considered as half that amount of energy, which can be thought of as equalizing the populations leaving no magnetization along the z axis. Thus far the QM concepts seem to agree with our macroscopic observations, and they can be helpful up to this point.

Magnetization Relaxation & Signal Detection

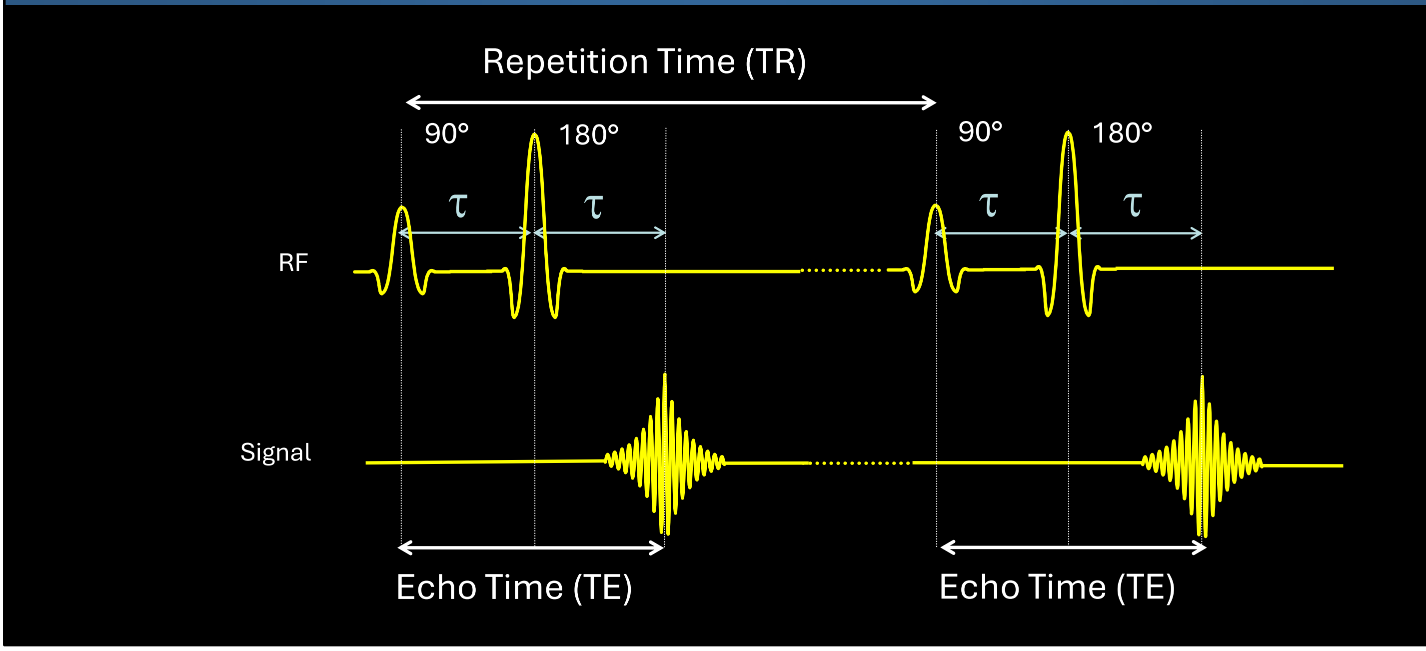


Relaxation is expressed in the Bloch equation. T_2 relaxation is caused by the irreversible interaction of individual spins. This means that the individual nuclei that make up the net magnetisation will start to precess at different rates in the transverse plane due to spin-spin (T_2) interactions as well as other nonuniformities in the static magnetic field, either intrinsic to the magnet or due to magnetic susceptibilities within the body, these have a time constant given by T_2'. The overall relaxation time is the sum of these relaxivities, i.e.,

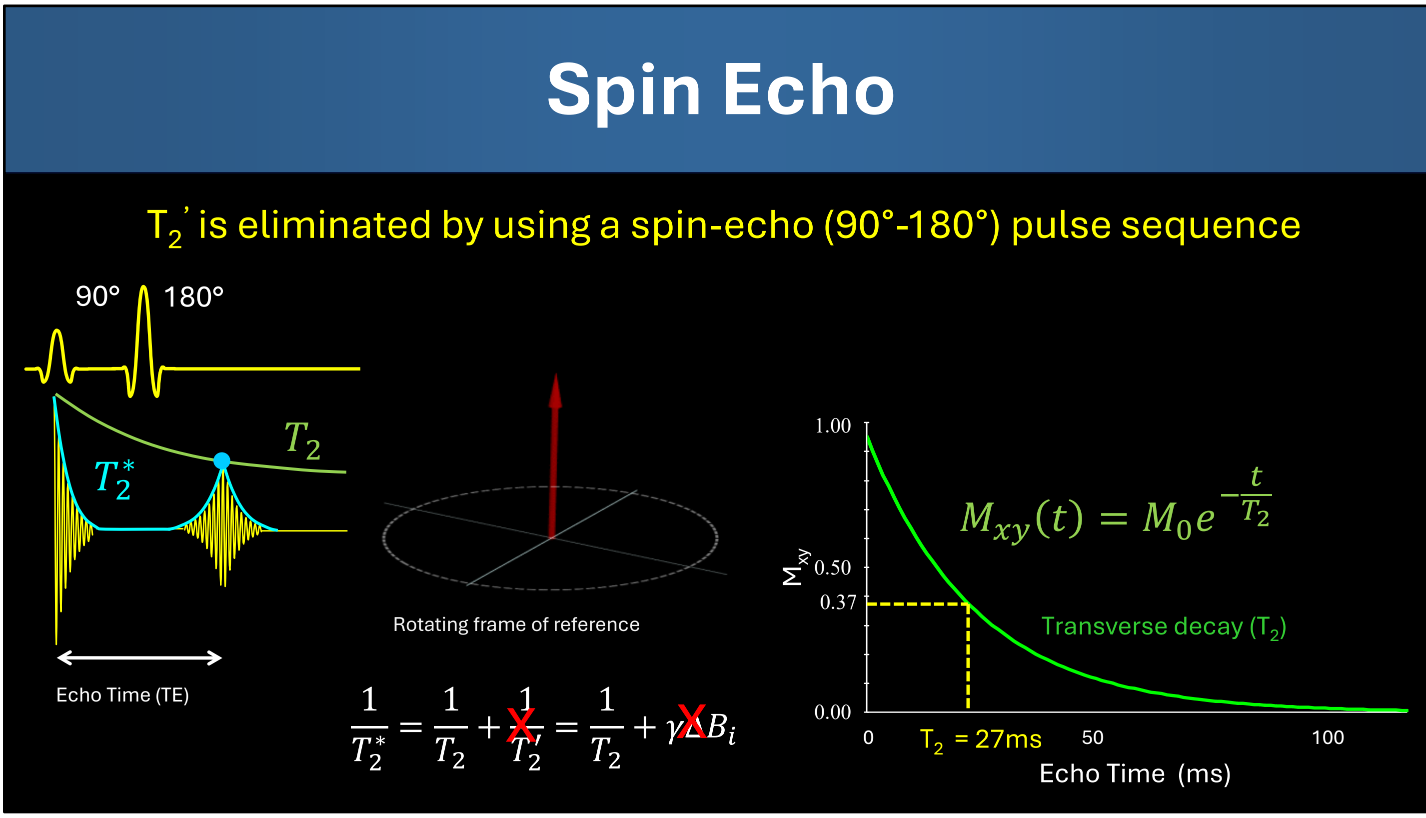
$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'} = \frac{1}{T_2} + \gamma \Delta B_i.$$

In the laboratory frame we can see some nuclei precessing faster (yellow arrows) and some precessing slower (white arrows) than the nominal Larmor frequency. The net magnetisation (cyan arrow) decreases in amplitude as the nuclei dephase with respect to each other in the transverse plane. The induced voltage is known as the free induction decay (FID)

Spin Echo



The most basic pulse sequence used in MRI is the spin echo. A spin echo involves two RF pulses, we will ignore the spatial localization gradients for the moment. We have a 90°excitation pulse followed a time τ later by a 180° refocusing pulse. After another period τ the magnetization will refocus producing an echo. The time from the 90° excitation pulse to the centre of the echo is known as the echo time (TE). We need to repeat the spin echo sequence multiple times in order acquire our image and the time from applying one 90° pulse to a subsequent 90° pulse is known as the repeat time or the repetition time (TR).



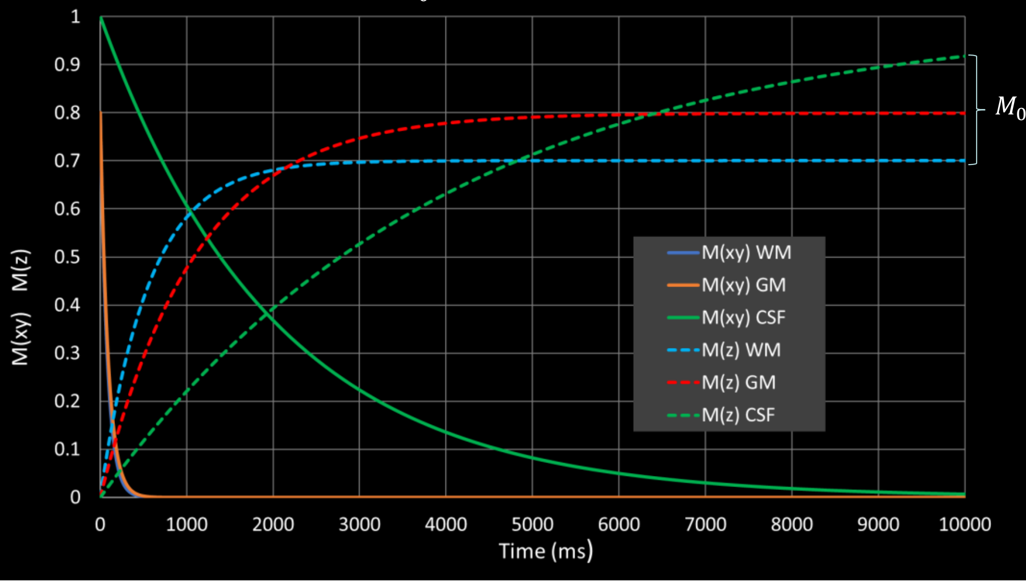
The advantage of the spin echo sequence is that it can eliminate T_2' dephasing. It achieves this using a 180° refocusing pulse. Following the 90° excitation the magnetisation starts to dephase in the transverse plane. We apply the 180° pulse and the magnetisation that was previously dephasing will now naturally rephase because of this phase flip resulting in the formation of an echo signal. There is some reduction in the echo amplitude due to T_2 decay since that is an irreversible, whereas the static field non uniformities that give rise to T_2' will not change over the time scale of the experiment, i.e., the degree of dephasing is matched by an identical amount of rephasing. In this case we can see that for this curve the T_2 relaxation time characteristic of the curve is 27ms, that is the time for the magnetisation to decay to 37% of its initial value.

T_1 and T_2 Relaxation (1.5T)

Tissue	T_1 (ms)	T_2 (ms)
Fat (adipose)	200	70
Liver	570	40
Muscle	1100	30
White matter	560	80
Grey matter	1100	90
CSF	4000	2000

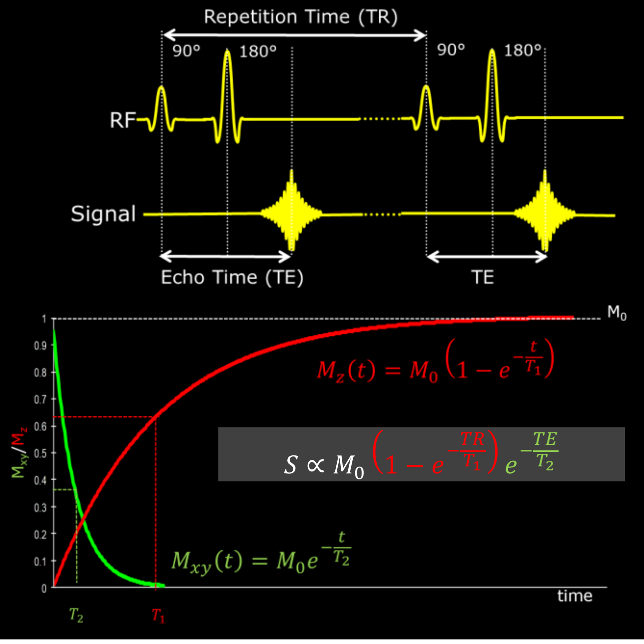
Approximate values

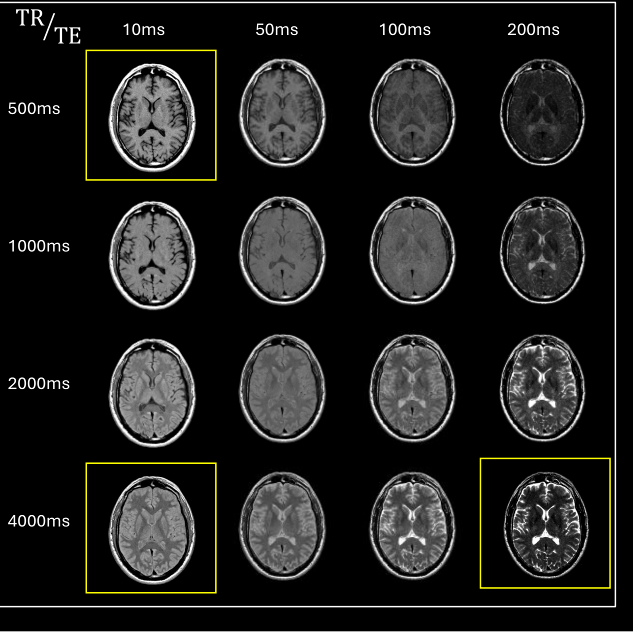
$$S \propto M_0 \left(1 - e^{-\frac{TR}{T_1}}\right) e^{-\frac{TE}{T_2}}$$



The contrast in a spin echo sequence is controlled by the TE and the TR. The TE controls the amount of T_2 dephasing, i.e., T_2 weighting in the image, whilst the TR controls the amount of T_1 recovery, i.e., T1-weighting in the image. We describe images as “weighted” since it is extremely difficult to have either zero TE to eliminate T_2 or very long TR, e.g., at least five times the longest T_1, to eliminate T_1. In any case the underlying proton density differences are always present. A proton density (PD) weighted image therefore has a long TR to reduce T_1 effects and a short TE to minimise T_2 effects. A T_2-weighted image has a long TR to minimise T_1 effects but a long TE to maximise T_2 contrast. A T_1-weighted image has a short TR to maximise the differences in T_1 relaxation but a short TE to minimise T_2 effects. Since it is not possible to directly observe the longitudinal recovery, the magnetisation must be tipped into the transverse plane to detect any signal. There will be inevitable T_2 decay during There will be a minimum TE and hence always some T_2 decay in T_1 or PD-weighted images.

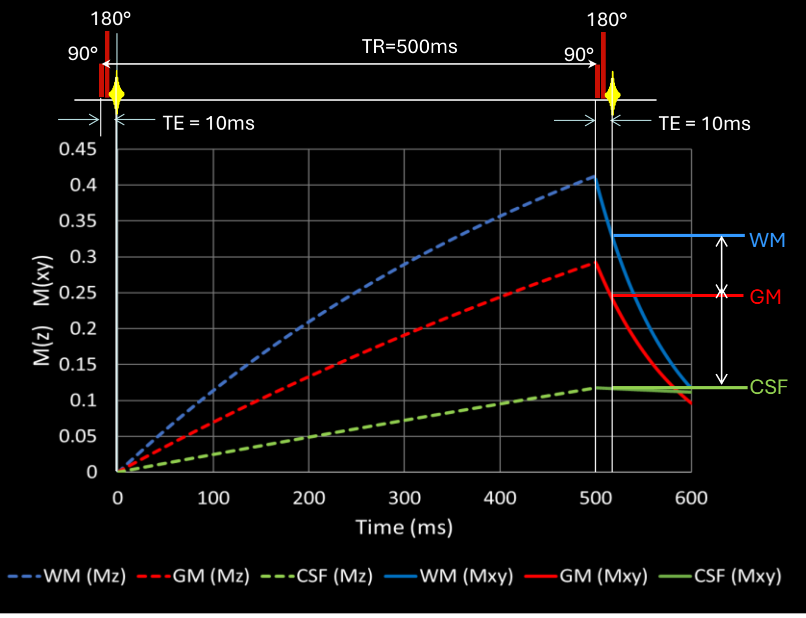
Spin Echo Contrast



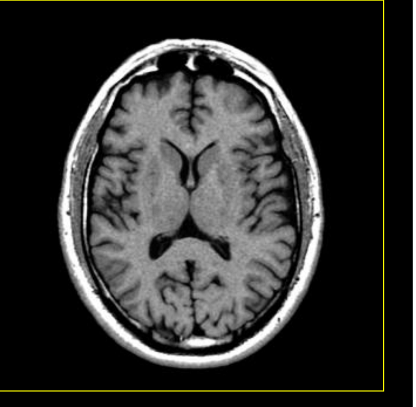


Although we have yet to discuss the mechanism for creating images it might be quite useful just to have a look at what the effect of these timing parameters and relaxation times are on a spin echo image. In the montage TE increases towards the right and TR increases downwards. A T_1-weighted image is shown at the top left, a proton density weighted image at bottom left and a T_2-weighted image at bottom right. These are the main contrast weightings used in clinical MRI.

T_1w Image

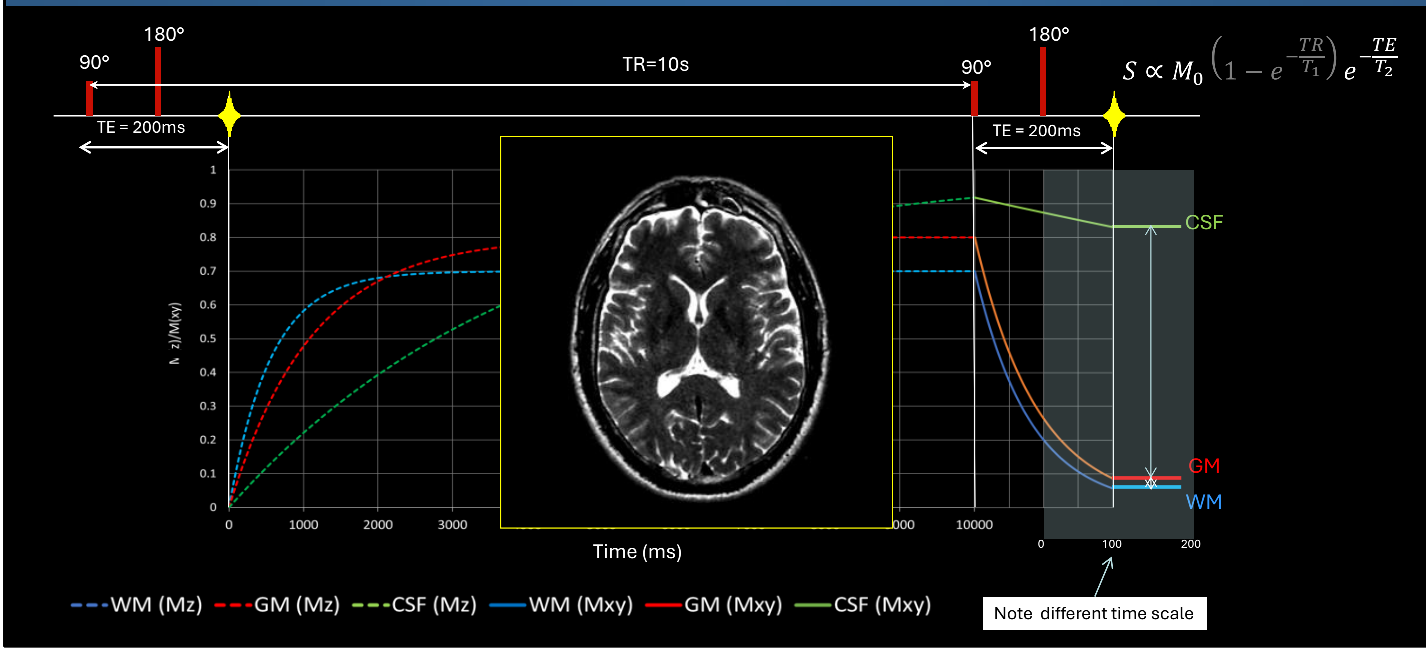


$$S \propto M_0 \left(1 - e^{-\frac{TR}{T_1}}\right) e^{-\frac{TE}{T_2}}$$

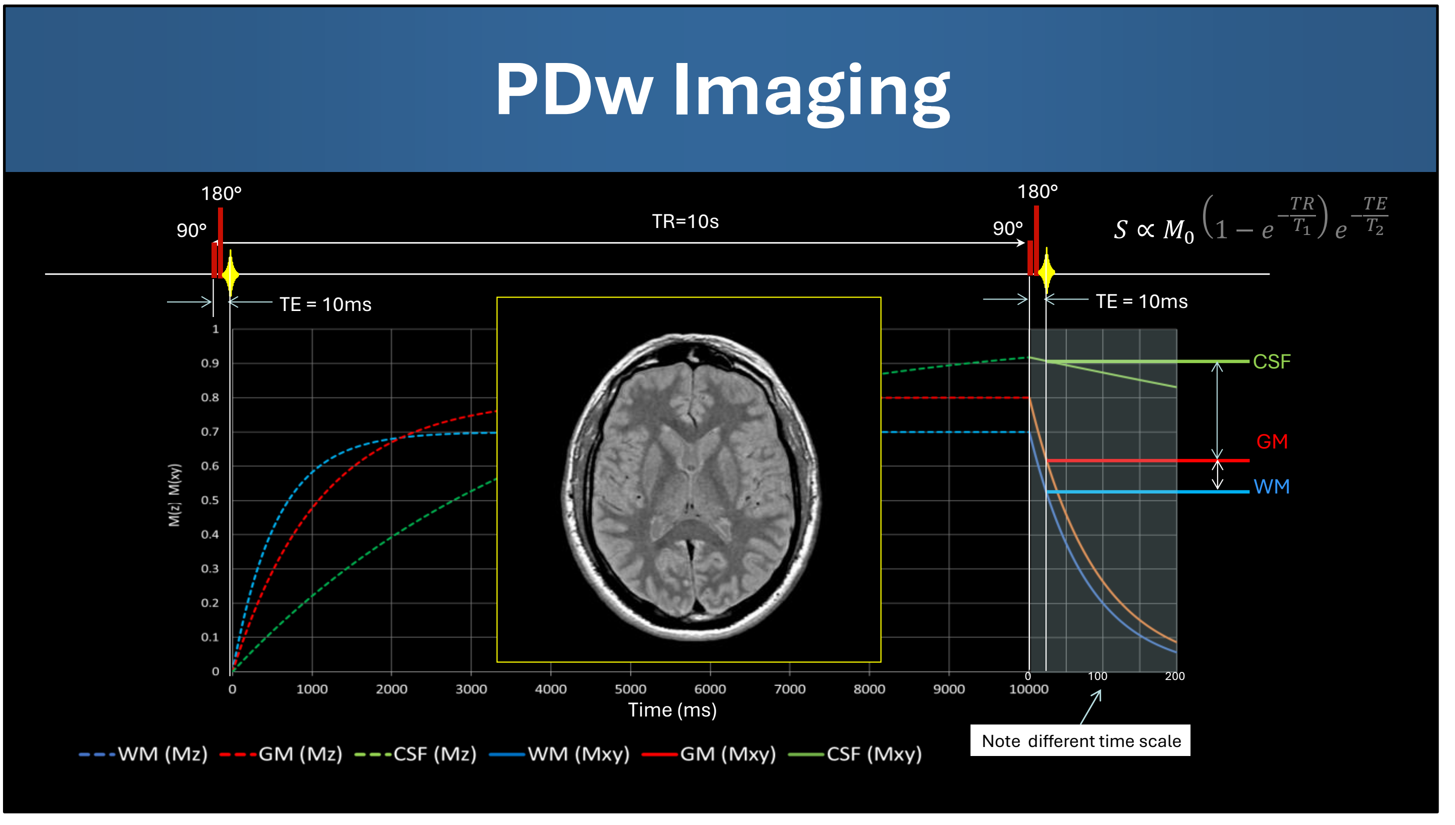


In T_1-weighted imaging we have a short TE to minimise T_2-weighting, but we also have a relatively short TR so that the magnetization will have partly recovered due to T_1 relaxation. TR is chosen so that the signal intensity differences due to T_1 recovery are emphasised. Note how the T_2 decay tends to reduce the signal intensity differences hence it is important to minimise T_2 decay effects using a short echo time. The white matter is brighter than the grey matter and the CSF in the ventricles and around the brain is darkest. In this image we have a minimum echo time of 10ms and a TR of 500 ms.

T_2w Imaging

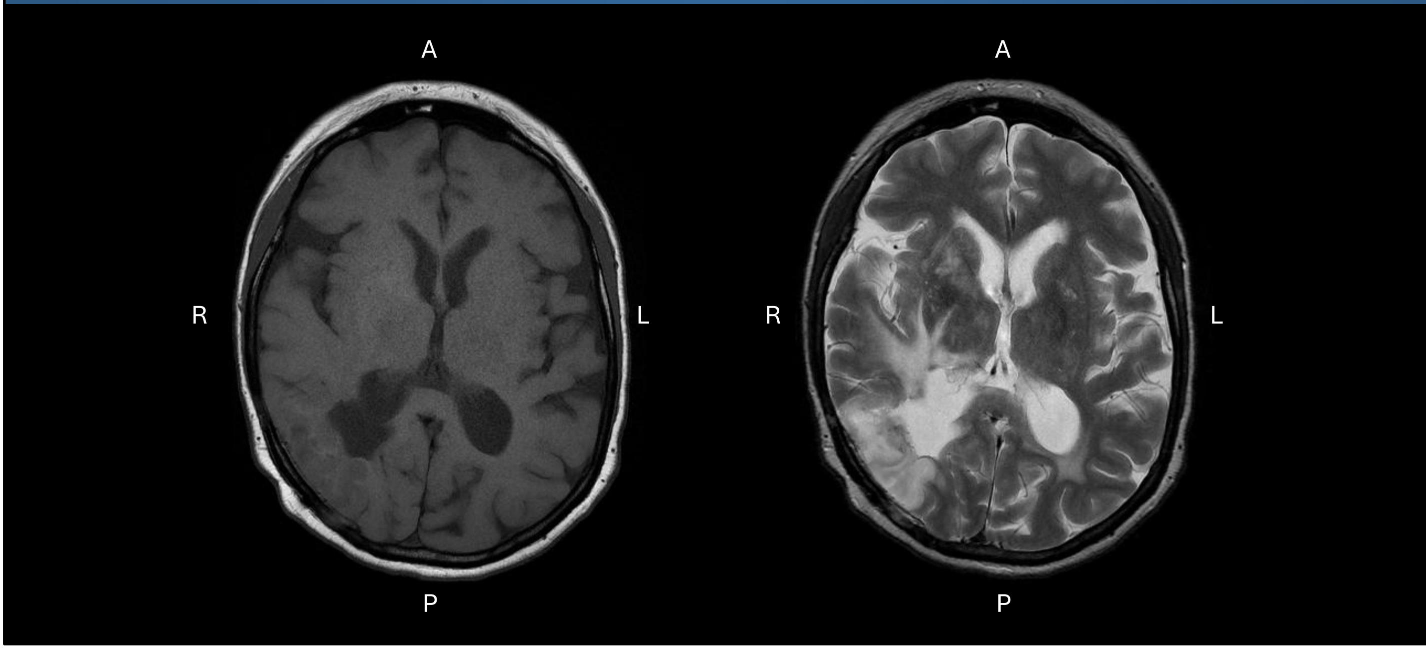


In T_2-weighted imaging we have a very long TR so that all the magnetization will have recovered due to T_1 relaxation, but with a long TE to demonstrate the signal due to differences in T_2 relaxation time. CSF has the brightest signal and very slightly higher signal from grey matter compared to white matter. Many pathologies will increase the T_2 relaxation time so we will see them as hyperintense on T_2w imaging This image was acquired with an echo time of 200 ms and a TR of 7 s.



In proton density weighted imaging we have a very short TE to minimise the effects of T_2 decay and a very long TR so that all the magnetization will have recovered due to T_1 relaxation and therefore we are left with signal differences which represents the relative proton densities of the tissues. Hence the CSF should have the brightest signal, but since the image was acquired using a TR of less than 10 s, due to the acquisition time, the CSF is relatively isointense with the grey matter. However, the white matter clearly demonstrates the lowest signal due to its reduced proton density. In this image we have an echo time of 10 ms and a TR of 7 s.

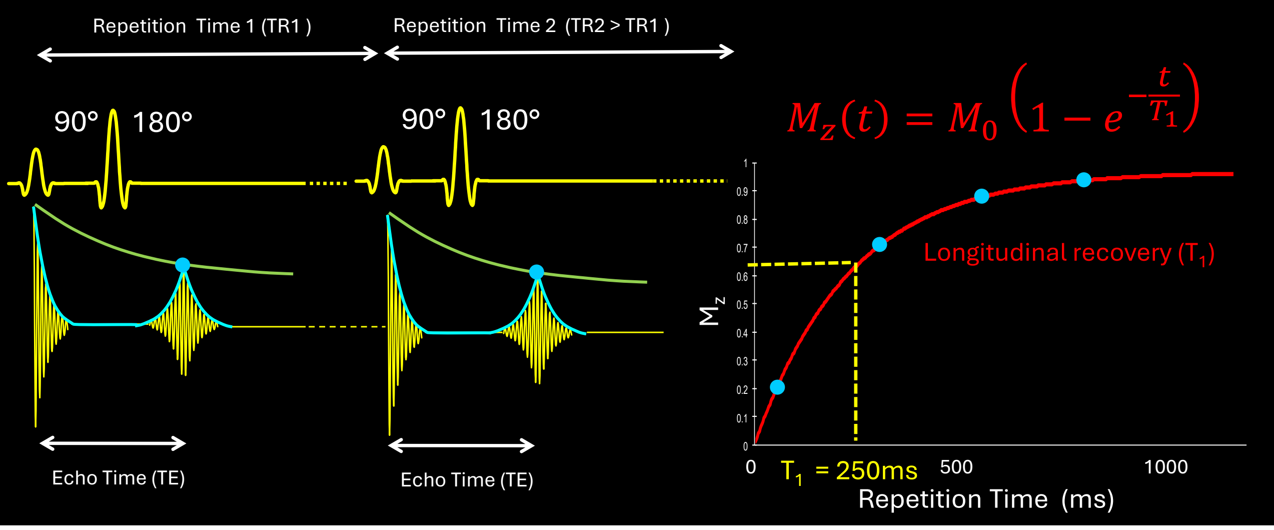
T_1w vs. T_2w Spin Echo



T_1w and T_2w images at matched locations in a patient with a brain tumour. The T_1w image does not demonstrate much image contrast although we can clearly see left-right asymmetry. On the T_2w image we can clearly see hyperintense regions on the right-hand side of the brain (MRI images are usually viewed as if you are looking up towards the patient's feet). Hence the increase in relaxation times normally seen in pathology appear brighter on T w imaging. Whereas the T_1w sequence is used more for anatomy.

Measuring T_1 Relaxation

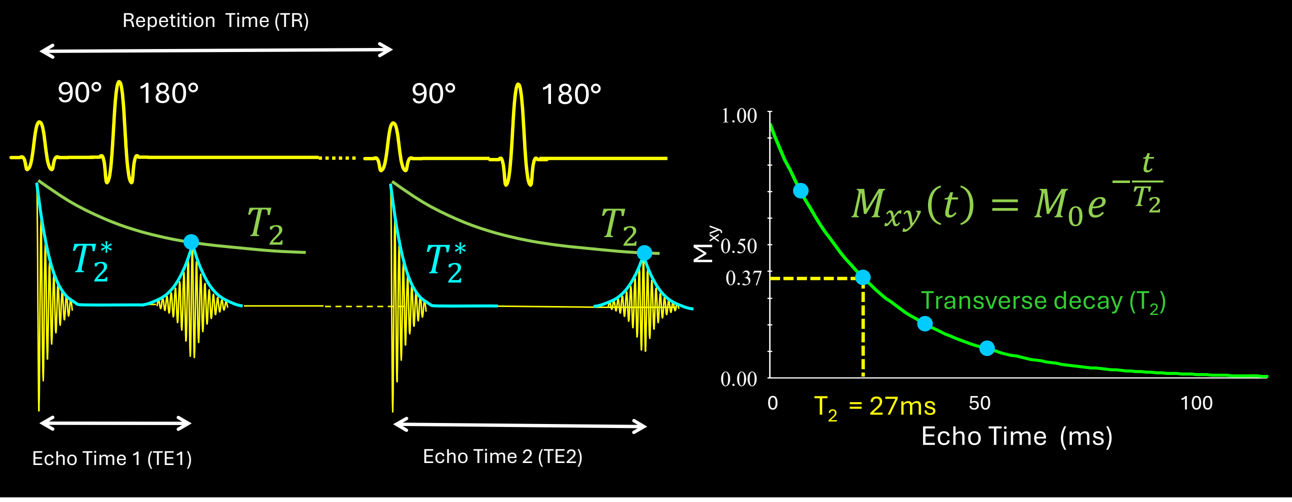
T_1 can be calculated from images acquired at different TRs but the TE needs to be minimised



It is possible to calculate the T_1 relaxation constant by acquiring signals with different TRs. Therefore, the TE needs to be kept as short as possible to minimise T_2 decay on the measured signals. The blue circles show the transverse magnetisation at different TRs from which the T1 relaxation time can be derived by fitting the signal equation to the signal values. In this case we can see that for this curve the T_1 relaxation time characteristic of the curve is 250 ms, that is the time for the magnetisation to recover to 63% of its initial value. Like T_2, it can be quite time consuming to make T_1 relaxation measurements, however there are several advanced pulse sequences that try to do this more rapidly and better.

Measuring T_2 Relaxation

T_2 can be calculated from images acquired at different TEs but the TR needs to be maximised



Although it is not generally performed clinically, we can measure the T_2 relaxation time on a pixel-by-pixel basis by acquiring images with different TEs and then we can fit the T_2 decay model to the measured data points. This allows us to estimate the T_2 relaxation time as well as the relative proton density. We need to ensure that we have a very long repetition time to make sure that the magnetization is fully back to equilibrium before we perform our next excitation with a different TE. Therefore, it can be quite time consuming to make T_2 relaxation measurements.

Learning Outcomes

- After this lectures you should be able to:
  - Explain how nuclear spin gives rise to magnetic resonance
  - Understand the principles of T_1, T_2 and T_2^*relaxation
  - Explain the principles of MR image formation
  - Describe the spin echo and gradient echo pulse sequences
  - Outline the basic components of an MRI system
  - Understand the safety issues related to MRI

Motion in the Presence of an Applied Field

- Net nuclear magnetisation $\mathbf{M}$ will precess about $\mathbf{B}$ at a frequency $\omega = \gamma B$ the motion is given by

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}$$

- Defining $\mathbf{M} = \hat{\imath} M_x + \hat{\jmath} M_y + \hat{k} M_z$ and expressing the cross product as the formal determinant

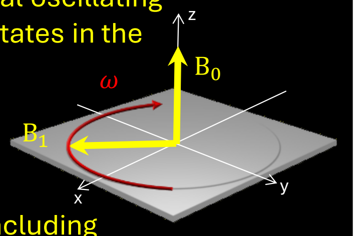
$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} = \gamma \det \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ M_x & M_y & M_z \\ B_x & B_y & B_z \end{bmatrix}$$

- Using Sarrus's rule, we get each component

$$\begin{bmatrix} dM_x/dt \\ dM_y/dt \\ dM_z/dt \end{bmatrix} = \gamma \begin{bmatrix} \hat{\imath}\left(M_y B_z - M_z B_y\right) \\ \hat{\jmath}\left(M_z B_x - M_x B_z\right) \\ \hat{k}\left(M_x B_y - M_y B_x\right) \end{bmatrix}$$

- In addition to the static field $\mathbf{B_o}$ we introduce an an additional oscillating magnetic field $\mathbf{B_1}$ that rotates in the x-y plane at frequency ω

$$B_x = B_1 \cos(\omega t)$$
$$B_y = -B_1 \sin(\omega t)$$
$$B_z = B_0$$



Substituting into $\frac{d\mathbf{M}}{dt}$ and including relaxation we get the Bloch equations

$$\frac{dM_y}{dt} = \gamma\left(M_y B_0 + M_z B_1 \sin(\omega t)\right) - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \gamma(M_z B_1 \cos(\omega t) - M_x B_0) - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = -\gamma(M_x B_1 \sin(\omega t) + M_Y B_1 \cos(\omega t)) - \frac{(M_z - M_0)}{T_1}$$

If we temporarily ignore relaxation the left-hand side of the slide demonstrates the Bloch equation for free precession about the static magnetic field along z. By defining M as a vector and expressing the cross product as the formal determinant and using Sarrus's rule we can derive expressions for the three components of magnetisation. Note that the change in the z-component of the magnetisation is unaffected by the z-component of the static field, this means that B_z has no effect on T_1 relaxation. Similarly, we can see that that all three components of the static field affect the transverse or T_2 relaxation.

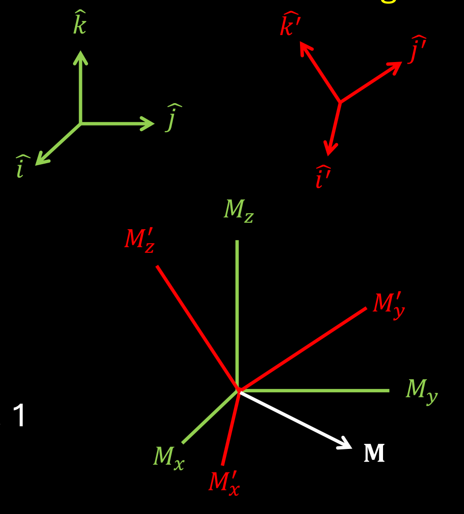
On the right-hand side, if we now introduce an RF pulse, i.e., an additional oscillating magnetic field B_1 that rotates in the transverse plane at a frequency of ω, and we now make B explicitly to be B_0 we have the differential equations for motion in the presence of an applied B_1 field. If we include relaxation these equations are generally referred to as the Bloch equations (plural) in the laboratory or fixed frame of reference.

Rotating Frame of Reference I

- We need to derive the equations of motion for the components of $\mathbf{M}$ in the rotating frame compared to the laboratory or fixed frame

$\mathbf{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$ in the laboratory frame

$\mathbf{M} = M'_x\hat{i}' + M'_y\hat{j}' + M'_z\hat{k}'$ in the rotating frame



- The time derivative of $\mathbf{M}$ in the laboratory frame is

$$\frac{d\mathbf{M}}{dt} = \left(M_x\frac{d\hat{i}}{dt} + \hat{i}\frac{dM_x}{dt}\right) + \left(M_y\frac{d\hat{j}}{dt} + \hat{j}\frac{dM_y}{dt}\right) + \left(M_z\frac{d\hat{k}}{dt} + \hat{k}\frac{dM_z}{dt}\right)$$

- Since the axes are fixed in the laboratory frame

$$\frac{d\hat{i}}{dt} = \frac{d\hat{j}}{dt} = \frac{d\hat{k}}{dt} = 0 \text{ hence } \frac{d\mathbf{M}}{dt} = \frac{dM_x}{dt}\hat{i} + \frac{dM_y}{dt}\hat{j} + \frac{dM_z}{dt}\hat{k} \quad \text{Eq. 1}$$

- The time derivative of $\mathbf{M}$ in the rotating frame is

$$\frac{d\mathbf{M}}{dt} = \left(M'_x\frac{d\hat{i}'}{dt} + \hat{i}'\frac{dM'_x}{dt}\right) + \left(M'_y\frac{d\hat{j}'}{dt} + \hat{j}'\frac{M'_y}{dt}\right) + \left(M'_z\frac{d\hat{k}'}{dt} + \hat{k}'\frac{dM'_z}{dt}\right) \quad \text{Eq. 2}$$

It is conceptually much easier to work in the rotating frame of reference. This like trying to read the label of a 7" vinyl record on a turntable that is rotating at 45 rpm. However, if the turntable were large enough you could stand on it so that you were also rotating at 45 rpm and then it would be much easier to read the label. On this and the next two slides we go through the derivation of the equations of motion for the components of M in the rotating frame. If we consider two coordinate systems, Mx, My, Mz and M'_x, M'_y, M'_z whose axes are rotated relative to one another, but their origins coincide. If we introduce two sets of unit vectors with the same unprimed and primed axes, we can express the magnetisation vector **M** in terms of its components along either set of axes. Since the origins coincide a point is represented by the same vector **M** in both systems; only the components of **M** are different along the different axes. The time derivative of **M** in the laboratory frame is given by Eq. 1 and in the rotating frame by Eq. 2.

Rotating Frame of Reference II

- Since the LHS of Eq. 1 and Eq. 2 are identical, i.e. $\frac{d\mathbf{M}}{dt} = \frac{d\mathbf{M}}{dt}$

$$\frac{dM_x}{dt}\hat{i} + \frac{dM_y}{dt}\hat{j}\frac{dM_z}{dt}\hat{k} = \left(M_x'\frac{d\hat{i}'}{dt} + \hat{i}'\frac{dM_x'}{dt}\right) + \left(M_y'\frac{d\hat{j}'}{dt} + \hat{j}'\frac{dM_y'}{dt}\right) + \left(M_z'\frac{d\hat{k}'}{dt} + \hat{k}'\frac{dM_z'}{dt}\right)$$

- Regrouping terms on the RHS

$$\underbrace{\frac{dM_x}{dt}\hat{i} + \frac{dM_y}{dt}\hat{j} + \frac{dM_z}{dt}\hat{k}}_{\left(\frac{d\mathbf{M}}{dt}\right)_{lab}} = \underbrace{\frac{dM_x'}{dt}\hat{i}' + \frac{dM_y'}{dt}\hat{j}' + \frac{dM_z'}{dt}\hat{k}'}_{\left(\frac{d\mathbf{M}}{dt}\right)_{rot}} + \underbrace{M_x'\frac{d\hat{i}'}{dt} + M_y'\frac{d\hat{j}'}{dt} + M_z'\frac{d\hat{k}'}{dt}}_{\text{effects of rotation}}$$

Since the LHS of Eq. 1 and 2 are identical we obtain the following relationship following regrouping of terms.

Rotating Frame of Reference III

$$\underbrace{\frac{dM_x}{dt}\hat{i} + \frac{dM_y}{dt}\hat{j} + \frac{dM_z}{dt}\hat{k}}_{\left(d\mathbf{M}/dt\right)_{lab}} = \underbrace{\frac{dM'_x}{dt}\hat{i}' + \frac{dM'_y}{dt}\hat{j}' + \frac{dM'_z}{dt}\hat{k}'}_{\left(d\mathbf{M}/dt\right)_{rot}} + \underbrace{M'_x\frac{d\hat{i}'}{dt} + M'_y\frac{d\hat{j}'}{dt} + M'_z\frac{d\hat{k}'}{dt}}_{\text{effects of rotation}}$$

- Linear velocity = angular velocity x position vector, i.e., $\boldsymbol{V} = \boldsymbol{\Omega} \times \boldsymbol{r}$ and because $\boldsymbol{V} = \frac{dr}{dt}$, $\frac{dr}{dt} = \boldsymbol{\Omega} \times \boldsymbol{r}$ therefore $\frac{d\hat{i}'}{dt} = \boldsymbol{\Omega} \times \hat{i}'$ $\frac{d\hat{j}'}{dt} = \boldsymbol{\Omega} \times \hat{j}'$ $\frac{d\hat{k}'}{dt} = \boldsymbol{\Omega} \times \hat{k}'$

$$\left(\frac{d\mathbf{M}}{dt}\right)_{lab} = \left(\frac{d\mathbf{M}}{dt}\right)_{rot} + M'_x(\boldsymbol{\Omega} \times \hat{i}') + M'_y(\boldsymbol{\Omega} \times \hat{j}') + M'_z(\boldsymbol{\Omega} \times \hat{k}')$$

$$\left(\frac{d\mathbf{M}}{dt}\right)_{lab} = \left(\frac{d\mathbf{M}}{dt}\right)_{rot} + \boldsymbol{\Omega} \times \left(M'_x\hat{i}' + M'_y\hat{j}' + M'_z\hat{k}'\right)$$

- Since $\mathbf{M} = M'_x\hat{i}' + M'_y\hat{j}' + M'_z\hat{k}'$

$$\left(\frac{d\mathbf{M}}{dt}\right)_{lab} = \left(\frac{d\mathbf{M}}{dt}\right)_{rot} + \boldsymbol{\Omega} \times \mathbf{M}$$

To interpret the time derivative of the rotating unit vectors, think of them as a position vector. This shows the fundamental relationship between time derivatives for a coordinate system rotating at an angular velocity ω.

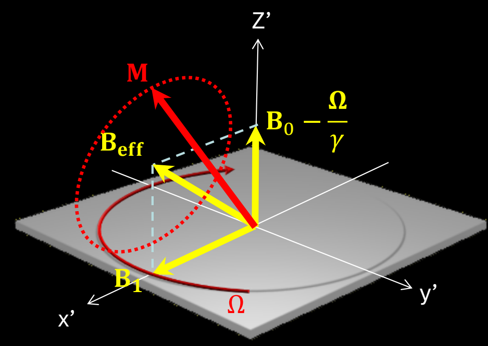
The Effective Field

- Since we know that $\left(\frac{d\mathbf{M}}{dt}\right)_{lab} = \gamma\mathbf{M} \times \mathbf{B}$ then $\gamma\mathbf{M} \times \mathbf{B} = \left(\frac{d\mathbf{M}}{dt}\right)_{rot} + \mathbf{\Omega} \times \mathbf{M}$
- Rearranging

$$\left(\frac{d\mathbf{M}}{dt}\right)_{rot} = \gamma\mathbf{M} \times \mathbf{B} - \mathbf{\Omega} \times \mathbf{M}$$

$$\left(\frac{d\mathbf{M}}{dt}\right)_{rot} = \gamma\mathbf{M} \times \mathbf{B} - \gamma\mathbf{M} \times \frac{\mathbf{\Omega}}{\gamma}$$

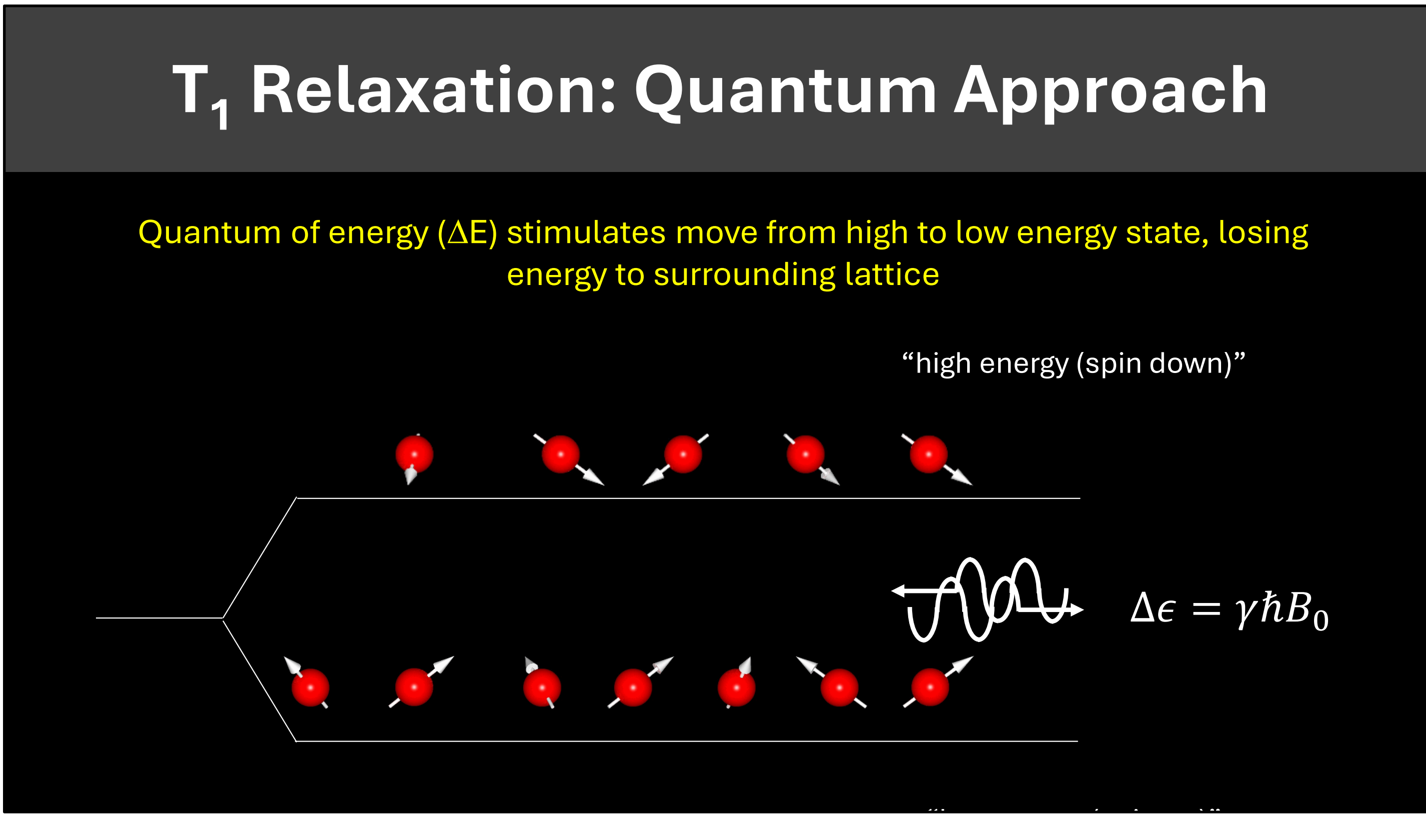
$$\left(\frac{d\mathbf{M}}{dt}\right)_{rot} = \gamma\mathbf{M} \times \underbrace{\left(\mathbf{B} - \frac{\mathbf{\Omega}}{\gamma}\right)}_{\text{effective field}} = \gamma\mathbf{M} \times \mathbf{B_{eff}} \quad \text{Eq. 3}$$



The effective field represents the difference in motion between the rotating frame and the laboratory reference frame. If we include $\mathbf{B}_1$, the effective field in a frame rotating at $\mathbf{\Omega}$ is given by $\mathbf{B_{eff}} = \left(\mathbf{B}_0 - \frac{\mathbf{\Omega}}{\gamma}\right) + \mathbf{B}_1$. If $\mathbf{\Omega} = \omega_0$ then $\mathbf{B}_1$ is static and $\mathbf{M}$ rotates about $\mathbf{B}_1$ $\left(\frac{d\mathbf{M}}{dt}\right)_{rot} = \gamma\mathbf{M} \times \mathbf{B}_1$ at a frequency of $\omega_1 = \gamma B_1$

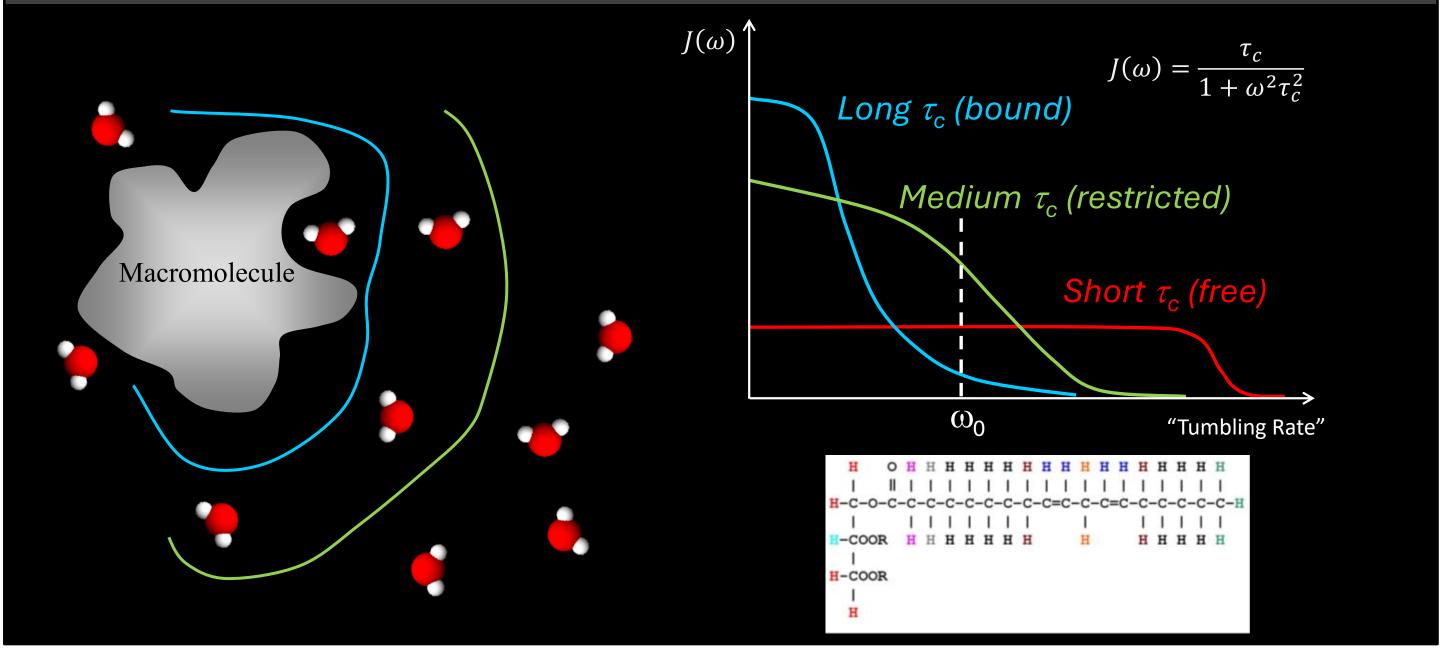
With a little more rearranging and substituting we obtain Eq. 3 whereby it is apparent that the magnetisation rotates about an effective magnetic field in the rotating frame just as it rotates about the applied magnetic field in the laboratory frame. If the frequency of the rotating frame equals the frequency of precession, then the magnetisation is static in the rotating frame.

If we now introduce an RF pulse to create a rotating B_1 field along the x-axis that adds to the effective field, we can see that if ω equals the Larmor frequency then B_1 is static, and the net magnetisation rotates about the x- axis at a rate equal to γB_1. Using the left-hand rule with your thumb pointing along the x-axis you can see that the rotation is clockwise about B_1. By changing the phase of B_1 it is possible to get the magnetisation to rotate about any axis. Note that if ω does not equal the Larmor frequency then the magnetisation will precess around the effective magnetic field and not into the transverse plane. Therefore, significant signal is only obtained when the frequency of the stimulus is the same or nearly the same as the natural vibration frequency of the magnetic resonance system. So, this classical mechanics approach agrees with the quantum mechanical view of resonance.



An RF pulse, on average, promotes protons from the low energy state to the high energy state causing a net absorption of energy. T_1 relaxation is the loss of the extra energy from the spin system to the surrounding environment, or 'lattice' (hence 'spin-lattice' relaxation time). However, the high energy state is a stable position for the proton, and it does not return to the lower state spontaneously but requires an external stimulating field. Since the external B_1 field has been switched off, the simulating field comes from inter and intra-molecular dipole interactions

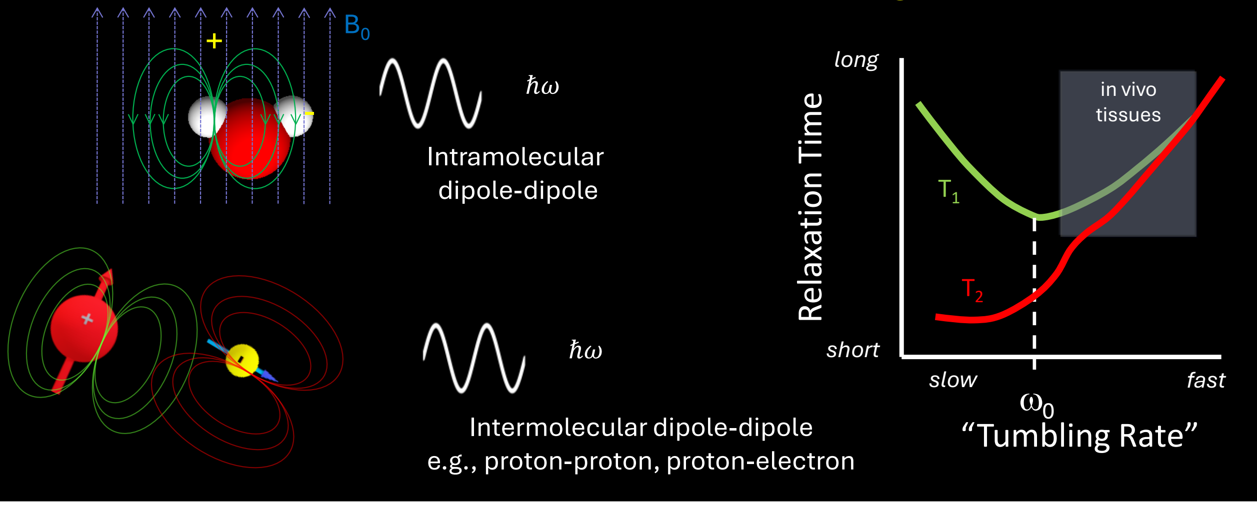
Molecular Tumbling



T_1 relaxation will primarily arise through the magnetic moment that one hydrogen nucleus 'sees' as it tumbles relative to the moment of the other hydrogen nucleus. This is often called an intra-molecular dipole-dipole interaction. However, nuclei must tumble at the Larmor frequency to induce the transitions. In vivo water, with its small molecular size, tumbles at a wide range of frequencies and therefore only a small fraction will ever tumble at the right frequency to induce T_1 relaxation. T_1 values are therefore longer for free water than for any other substance in the body (approximately 4000 ms at 1.5T). When a water is molecule comes close enough to a large macromolecule, its tumbling may be slowed to a rate much closer to the Larmor frequency. Water molecules with restricted motion frequently occur in biological systems where their molecular motion is slowed by interaction with polar amide and hydroxyl groups on the surfaces of macromolecules. The T_1 value of "bound" or "structured" water is therefore much shorter than that of free water (typically about 400-800 ms at 1.5 T). This is the type of water whose signal we primarily measure when imaging solid organs like the liver or brain. One way to characterize the motion of a molecule is by its correlation time *(τ_c)*, the time required to rotate by approximately 1 radian (~57°). As expected, smaller molecules rotate faster (and have smaller *τ_c's)* than larger ones. The correlation time also decreases with increasing temperature (kinetic energy causes them to rotate faster). As a rule of thumb used by chemists, the rotational correlation time (in nanoseconds) of a molecule in aqueous solution at room temperature is about half its molecular weight (in kilodaltons). Because of random thermal fluctuations and collisions, otherwise identical molecules with the same correlation times *(τ_c)* will rotate at different angular velocities (ω). The probability of finding a molecule rotating at a given frequency is proportional to the spectral density function J(ω). For spherical molecules in solution the form of the spectral density is expressed by the equation above. For large molecule, like lipids, their tumbling rate is just right to result in efficient T_1 relaxation, hence why fat has the shortest natural T_1.

T_1 Relaxation Mechanism

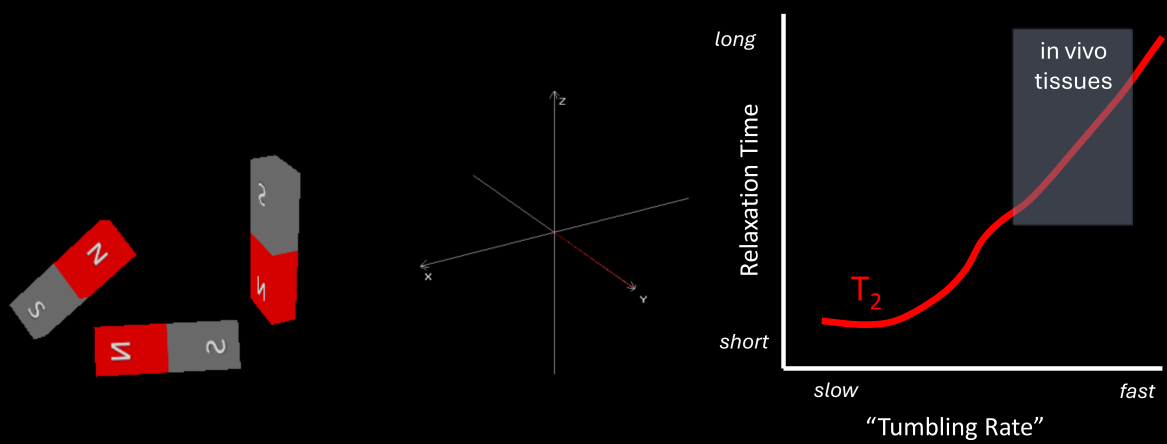
As molecules tumble/vibrate the protons are exposed to an alternating magnetic field that stimulates relaxation. T_1 relaxation is dependent upon the molecular motion and tumbling rate



T_1 relaxation requires a stimulating fluctuating magnetic field. The fluctuations come from neighbouring protons or other nuclei or molecules, which have magnetic moments. In water the nearest adjacent nucleus will be the other hydrogen atom on the same molecule. Therefore, relaxation will primarily arise through the magnetic moment that one hydrogen nucleus 'sees' as it tumbles relative to the moment of the other hydrogen nucleus. This is often called an intra-molecular dipole-dipole interaction. Similarly, intermolecular dipole interaction such as between proton-proton and proton-electron interactions also contribute to T_1 relaxation.

T_2 Relaxation Mechanism

Magnetic moments of neighbouring protons alter local magnetic field causing a loss of phase coherence (spin-spin relaxation)



T_2 relaxation arises from the exchange of energy between spins, hence the term 'spin-spin' relaxation. No energy is lost from the spin system, but the decay of transverse magnetization arises from the loss of phase coherence between spins, which arises from magnetic field nonuniformities. These inhomogeneities may be either intrinsic or extrinsic, i.e., internal to the proton system or external in the scanner. Only the intrinsic inhomogeneities contribute to T_2. A description of molecular motions can also be used to describe the mechanism of T_2 relaxation. When molecules are tumbling very rapidly, i.e., we can consider them "free" and not "bound" to surrounding macromolecules then a particular spin dipole, i.e., bar magnet, will see the local magnetic field as fluctuating very rapidly and effectively averaging out over a few milliseconds. This results in a relatively uniform local field and little dephasing and is sometimes termed 'motional averaging'. Conversely a slowly tumbling molecule (bound protons close to large molecules) will see a relatively static magnetic field nonuniformity and will be more effectively dephased, i.e., will have a shorter T_2.